

Comparison of $4n+1$ and $4n-1$ Primes

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Abstract

The famous mathematician Littlewood proposed an unsolved conjecture in number theory, which has two descriptions :1. Before any number in front of K , the number of prime numbers in the form of $4n+1$ is no more than $4n-1$; 2. After this number k , the number of prime numbers in the form of $4n-1$ is no more than the number of primes in the form of $4n+1$. This paper proves that there is a conclusion about this conjecture: 1. There is a critical point K , and the number of $4n+1$ primes before any natural number in front of k is not more than $4n-1$. 2. There is no critical point, the number of $4n+1$ primes is never more than $4n-1$ primes. One of the two conclusions must be true.

Keywords: Number Theory; $p=4n+1$; $p=4n-1$

INTRODUCTION

There is an unsolved conjecture in number theory[1] which was put forward by Littlewood [2]:1. Before any number in front of K , the number of prime numbers in the form of $4n+1$ is no more than $4n-1$; 2. After this number k , the number of prime numbers in the form of $4n-1$ is no more than the number of primes in the form of $4n+1$; This paper proves that there is a conclusion about this conjecture: 1. There is a critical point K , and the number of $4n+1$ primes before any natural number in front of k is not more than $4n-1$. 2. There is no critical point, the number of $4n+1$ primes is never more than $4n-1$ primes. One of the two conclusions must be true.

Lemma 1[5]: If $N = 2mn+m+n$, then $2N+1$ is a composite number

Proof: If $N = 2mn+m+n$, then $2N+1 = 2(2mn+m+n)+1 = 4mn+2m+2n+1 = (2m+1)(2n+1)$, so $2N+1$ is a composite number. If $N \neq 2mn+m+n$, then $2N+1$ must be a prime number.

Lemma 2: If $N = 8ab+6a+2b+1$, then $2N+1$ is a composite number

Proof: For $2mn+m+n$ is odd numbers, to let $m=2a$, $n=2b+1$, $2mn+m+n = 2(2a)(2b+1)+2a+2b+1 = 8ab+6a+2b+1$, so if $N = 8ab+6a+2b+1$, it is just $N = 2mn+m+n$, then $2N+1$ is a composite number, if $N \neq 2(4ab+3a+b)+1$, then $2N+1$ is a prime number.

Lemma 3: If $N = 8ab+2a+2b$, then $2N+1$ is a composite number; If $N = 8ab+6a+6b+4$, then $2N+1$ is a composite.

Proof: For $2mn+m+n$ is even numbers, to let $m=2a$, $n=2b$, $2mn+m+n = 2(2a)(2b)+2a+2b = 8ab+2a+2b$, so if $N = 8ab+2a+2b$, it is just $N = 2mn+m+n$, then $2N+1$ is a composite number, if $N \neq 8ab+2a+2b$, then $2N+1$ is a prime number.

For $2mn+m+n$, to let $m=2a+1$, $n=2b+1$, $2mn+m+n = 2(2a+1)(2b+1)+2a+1+2b+1 = 8ab+6a+6b+4$, so if $N = 8ab+6a+6b+4$, it is just $N = 2mn+m+n$, then $2N+1$ is a composite number, if $N \neq 8ab+6a+6b+4$, then $2N+1$ is a prime number.

Theorem: Before a number k , the number of prime numbers in the form of $4n-1$ is more than that of $4n+1$.

Proof: $4n+1 = 2(2n)+1$, it is the 2 times of even; $4n-1 = 2(2n-1)+1$, it is the 2 times of odd. For $2mn+m+n$, let $m=2a$, $n=2b$, then $2mn+m+n = 8ab+2a+2b$; let $m=2a+1$, $n=2b+1$, then $2mn+m+n = 8ab+6a+6b+2$; let $m=2a$, $n=2b+1$, then $2mn+m+n = 8ab+6a+2b+1$. N_1 , N_2 , and N_3 respectively represent the set of positive integers that satisfy the following categories (k is a positive integer).

$$8ab+2a+2b \leq (k-1)/2, N1$$

$$8ab+6a+6b+2 \leq (k-1)/2, N2$$

$$8ab+6a+2b+1 \leq (k-1)/2, N3$$

Before k , the number of prime numbers in the form of $4n-1$ is $n1$, prime numbers in the form of $4n+1$ is $n2$. According to lemma 3, before k , the number of composite numbers in the form of $4n-1$ is $n3$, the number of composite numbers in the form of $4n+1$ is $n4$. $N3$ is the number of $n3$, $N1+N2$ is the number of $n4$, then $n1=k/2-n3$, $n2=k/2-n4=k/2-(N1+N2)$.

Due to the form $8ab+6a+2b+1$ is bigger than $8ab+2a+2b$, so for k , before $(k-1)/2$, $N3 \leq N1+N2$, so before k , $n1 \geq n2$.

Note: the numbers that can be expressed of $8ab+6a+2b+1$, $8ab+6a+6b+2$ and $8ab+2a+2b$, there's a repetition in themselves, and the repetition numbers of $8ab+2a+2b$ is more than $8ab+6a+2b+1$ and $8ab+6a+6b+2$. For repeated interference effect, there will be such a phenomenon: before k , even if there are repeated, and $8ab+6a+2b+1$ shows natural number less, since more and more repeat after more than k , the interference is very serious, repeated more than the expression showing gap between the numbers, it goes the other way around. Is the degree of repetition greater than the difference in the expression that represents the natural number? If so, it can get conclusion 1: there is a critical point K , and that $4n+1$ primes are not more than $4n-1$ before any natural number in front of K . If not, get conclude 2: There is no critical point and the number of $4n+1$ primes is never more than $4n-1$ primes. One of the two conclusions must be true.

CONCLUSION

1. There is a critical point K , and the number of $4n+1$ primes before any natural number in front of k is not more than $4n-1$.

2. There is no critical point, the number of $4n+1$ primes is never more than $4n-1$ primes.

One of the two conclusions must be true.

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