

Calculation and Analysis of Bearing Capacity of Double-Layer Foundation Based on Prandtl Assumption

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Abstract

Based on the basic concept of Plandell's assumption, the plasticity balance principle is applied to analyze the bearing capacity of the two-layer foundation, and the formula of bearing capacity of the two-layer foundation is obtained. The variation of bearing capacity of the foundation under different soil conditions is analyzed, and different calculation parameters are obtained. The effect on the ultimate bearing capacity of the two-layer foundation. The study found that: (1) For homogeneous soils, the formula obtained in this paper is the same as the Plandell formula; (2) the cohesive force of the soil, the soil bulk density above the foundation, and the aspect ratio have less influence on the bearing capacity of the foundation than the internal friction angle. Much; (3) The upper and lower layers of different strength ratios have different critical depths and increase with the increase of the relative strength of the upper layers. (4) This paper assumes that the foundation is completely smooth and ignores the influence of the bulk density of the foundation soil, which is not consistent with sand.

Key words: *Double Layered Foundation; Bearing Capacity of Foundation; Prandtl; Internal Friction Angle*

1 INTRODUCTION

The bearing capacity of the foundation refers to the ability of the foundation to withstand the ultimate load. In many classical theories of soil mechanics, many calculation formulas for ultimate bearing capacity are provided, such as the limit bearing formula of Taishaji and Hansen. However, most of these calculation formulas are for homogeneous foundations and are not applicable to layered soil foundations. In fact, the foundation is usually a heterogeneous layered soil, and the differences in mechanical properties between the soil layers are large. The failure mode of the layered soil foundation and the failure mode of the homogeneous soil foundation are often very different. It is impossible to form the shape of the sliding surface when the homogeneous soil foundation is formed, and the corresponding calculation method of the bearing capacity is bound to be very different.

For the calculation of the bearing capacity of the two-layer foundation, the predecessors have done a lot of research: Early Meyerhoff and Hansen et al.^[1] semi-theoretical and semi-empirical derivation of the two-layer foundation bearing capacity formula. Mahmoud Ghazavi, Amir Hossein Eghbali et al. Mahmoud Ghazavi, Amir Hossein Eghbali^[2] calculated the bearing capacity coefficient of shallow foundation on the double-layered granular soil section by the limit equilibrium method, calculated the equivalent bearing capacity coefficient and calculated the ultimate bearing capacity, and compared with the finite element numerical analysis. Based on the theory of soil limit equilibrium, Qiyi Zhang^[3] used the general finite element analysis software ABAQUS to obtain the bearing capacity and failure envelope of two layers of undrained foundation under horizontal and vertical loads. Mohammad Reza Lotfzadeh^[4] used the stress characteristic line method to derive the prediction algorithm for the static bearing capacity of strip foundations on two-layer foundations. However, the limit analysis method is subjectively affected by the assumption of failure modes and velocity fields. It is difficult to estimate the static field under complex conditions, and its solution range is limited. The improvement of the Meyerhoff-Hanna formula proposed by Yuan Fanfan^[5] to some extent satisfies the computational requirements of multi-layer foundations, but for most of the more complex foundations (for example, geological parameters into irregularly distributed block foundations) Still

powerless. G. G. MEYERHO^[6] studied the ultimate bearing capacity of foundations on two layers of foundations for both the soft clay and the hard clay. Different soil failure modes were analyzed and compared with the model test results of circular and strip foundations and some ground damage observations. Okamura^[7] and Lee^[8] carried out indoor model tests and centrifuge tests in different basic forms, which provided a reliable test basis for determining the ultimate bearing capacity of the two-layer foundation. However, the experimental operation is complicated, the size effect is obvious, and the similarity between the model and the actual soil layer affects the experimental results. Griffiths^[9]、Burd^[10] used conventional numerical methods such as finite element method and finite difference method to calculate the ultimate bearing capacity of sand-clay two-layer foundation, and analyzed the influence of different sand, clay strength and sand thickness on ultimate bearing capacity.

The theoretical method adopted by the above-mentioned double-layer foundation bearing capacity is difficult to be promoted in engineering because of its own complexity. Therefore, practical calculation methods are often used in practical engineering applications for design and analysis. Based on the basic concept of Prandtl's assumption, this paper analyzes the bearing capacity of two-layer foundation by applying the principle of plastic balance, and obtains the formula of bearing capacity of two-layer foundation.

2 CALCULATION AND ANALYSIS OF ULTIMATE BEARING CAPACITY OF DOUBLE-LAYER FOUNDATION

2.1 Hypothesis of Double-Layer Foundation Slip Line

The premise of using the slip line theory to analyze the ultimate bearing capacity of the foundation is to determine the approximate slip line shape. The two-layer foundation slip line field assumed in this paper (Fig. 1, only the right half of which is shown here, the left half is symmetrical with the right half) is based on Plandell's calculation of the bulk density of the foundation soil. However, according to the limit equilibrium theory^[11], if considering the bulk density of the foundation soil, the slip line of the plastic zone is not necessarily a straight line, nor can it be written in the form of a mathematical formula. Therefore, this paper ignores the influence of soil bulk density on the shape of the slip line. At this time, the soil under the base is divided into five zones: one I zone, (elastic core zone),two II zones, (Transition zone),and two III zones,(Passive Ranken area). The dividing line for each area is a straight line. For the two-layer foundation here, the physical and mechanical properties of the first layer of soil are c_1 、 φ_1 、 γ_1 ,The physical and mechanical properties of the second layer are c_2 、 φ_2 、 γ_2 . It is assumed that the straight line portion IN of the slip line is turned by the interface of the double-layered soil. In order to ensure that the IN runs through the subsoil, a precondition must be assumed:

$$\frac{h}{b} < \frac{1}{2} \cot \left(\frac{\pi}{4} - \frac{\varphi_1}{2} \right) \quad (1)$$

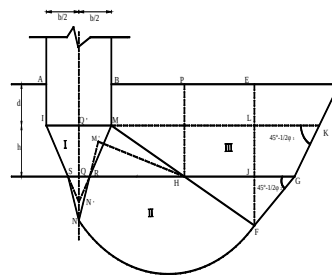


FIGURE 1 SKETCH MAP OF SLIP LINE FIELD ON DOUBLE LAYERED FOUNDATION

In order to derive the formula for the ultimate load, consider the force system acting on the soil QNFL, as shown in Figure 2:

$$\alpha_1 = \frac{\pi}{4} - \frac{\varphi_1}{2}, \alpha_2 = \frac{\pi}{4} - \frac{\varphi_2}{2}.$$

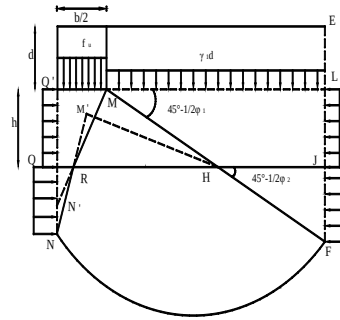


FIGURE 2 SKETCH MAP OF SOIL QNFL

2.2 Ultimate Bearing Capacity without Soil Bulk Density f_u

When the bulk density of the soil is not taken into account, the derivation process can be calculated by the method of solving the ultimate bearing capacity of the foundation

Uniform active earth pressure on QQ's surface:

$$P_{Q'Q} = f_u \tan^2 \alpha_1 - 2c_1 \tan \alpha_1 \quad (2)$$

Uniform active earth pressure on the Q'N surface:

$$P_{Q'N} = f_u \tan^2 \alpha_2 - 2c_2 \tan \alpha_2 \quad (3)$$

Passive earth pressure on the LJ surface:

$$P_{EJ} = q \cot^2 \alpha_1 + 2c_1 \cot \alpha_1 \quad (4)$$

Passive earth pressure on the JF surface:

$$P_{JF} = q \cot^2 \alpha_2 + 2c_2 \cot \alpha_2 \quad (5)$$

The moments acting on the soil unit Q'NFE for each force to point M are:

1. Torque generated by the ultimate bearing capacity on the QM surface:

$$M_1 = f_u \frac{b}{2} \cdot \frac{b}{4} = \frac{b^2}{8} f_u \quad (6)$$

2. The torque generated by the uniform earth pressure on the QN surface:

$$M_2 = \frac{h^2}{2} \left(f_u \tan^2 \alpha_1 - 2c_1 \tan \alpha_1 \right) + \left(f_u \tan^2 \alpha_2 - 2c_2 \tan \alpha_2 \right) \cdot l_{QN} \left(h + \frac{l_{QN}}{2} \right) \quad (7)$$

3. Uniform passive earth pressure on the LF surface

$$M_3 = \frac{h^2}{2} \left(q \cot^2 \alpha_1 + 2c_1 \cot \alpha_1 \right) + \left(q \cot^2 \alpha_2 + 2c_2 \cot \alpha_2 \right) \cdot l_{JF} \left(h + \frac{l_{JF}}{2} \right) \quad (8)$$

4. Torque generated by cohesion on the NF surface:

$$\begin{aligned}
M_4 &= \int_0^l c_2 ds \cdot \cos \cos \varphi_2 \cdot r \\
&= \int_0^{\pi/2} c_2 \left(r_0 e^{\theta \tan \varphi_2} \right)^2 d\theta \\
&= \frac{c_2 r_0^2}{2 \tan \varphi_2} \left(e^{\pi \tan \varphi_2} - 1 \right)
\end{aligned} \tag{9}$$

5. Torque generated by cohesion on the ML surface:

$$M_5 = \frac{1}{2} q \cdot l_{ML}^2 \tag{10}$$

According to $\sum M = 0$ gets:

$$M_1 + M_2 - M_3 - M_4 - M_5 = 0$$

$$\begin{aligned}
&f_u \left(\frac{b^2}{8} + \frac{h^2}{2} \tan^2 \alpha_1 + L_2 \tan^2 \alpha_2 \right) \\
&= q \left(\frac{h^2}{2} \cot^2 \alpha_1 + L_3 \cot^2 \alpha_2 + \frac{1}{2} l_{ML}^2 \right) \\
&+ c_1 h^2 \cot \alpha_1 + c_1 h^2 \tan \alpha_1 \\
&+ c_2 \left[2 \cdot L_3 \cot \alpha_2 + \frac{r_0^2}{2 \tan \varphi_2} \left(e^{\pi \tan \varphi_2} - 1 \right) + 2 \cdot L_2 \tan \alpha_2 \right] \\
\\
&f_u = q N_q + c_1 N_{c_1} + c_2 N_{c_2} \\
N_q &= \frac{\left(\frac{h^2}{2} \cot^2 \alpha_1 + \frac{1}{2} l_{ML}^2 + L_3 \cot^2 \alpha_2 \right)}{\left(\frac{b^2}{8} + \frac{h^2}{2} \tan^2 \alpha_1 + L_2 \tan^2 \alpha_2 \right)} \\
N_{c_1} &= \frac{\left(h^2 \cot \alpha_1 + h^2 \tan \alpha_1 \right)}{\frac{b^2}{8} + \frac{h^2}{2} \tan^2 \alpha_1 + L_2 \tan^2 \alpha_2} \\
N_{c_2} &= \frac{2 L_2 \tan \alpha_2 + 2 L_3 \cot \alpha_2 + \frac{r_0^2}{2 \tan \varphi_2} \left(e^{\pi \tan \varphi_2} - 1 \right)}{\frac{b^2}{8} + \frac{h^2}{2} \tan^2 \alpha_1 + L_2 \tan^2 \alpha_2} \\
L_2 &= l_{QN} (2h + l_{QN}) \\
L_3 &= l_{JF} (2h + l_{JF})
\end{aligned}$$

3 PARAMETER ANALYSIS OF BEARING CAPACITY OF DOUBLE-LAYER FOUNDATION

3.1 Comparison of the Method of This Paper with the Prandtl Method

Equation (21) is the bearing capacity formula calculated by Planter's theory.

$$q_u = (\gamma D + c \cdot \cot \varphi) e^{\pi \tan \varphi} \frac{1 + \sin \varphi}{1 - \sin \varphi} - c \cdot \cot \varphi \quad (11)$$

In order to verify the rigor of this paper, we now compare it with the Prandtl method which ignores the bulk density of the soil. Since the Prandtl method is suitable for homogeneous soil layers, it is assumed that the physical properties of the upper and lower layers are the same: $\varphi_1 = \varphi_2 = \varphi$; $c_1 = c_2 = c$; $\gamma_1 = \gamma_2 = \gamma$. At this time, it is consistent with the Prandtl formula (21).

3.2 Influence of Soil Parameters on Ultimate Bearing Capacity of Two-Layer Foundation

In view of the intricacies of practical problems, there are many factors affecting the ultimate bearing capacity of the foundation, and there is an interaction between these factors, and the experimental workload is large. In order to find out the main factors and secondary factors of these influencing factors, grasp the essence of the problem, in order to further discover the law. In this paper, the orthogonal experimental design method is used, and the "orthogonal table" is used to arrange the experiment reasonably. The number of trials can be used to determine the primary and secondary factors affecting the ultimate bearing capacity of the foundation.

It is assumed that the indicators of the upper layer soil are unchanged, and the influence of the lower layer soil on the ultimate bearing capacity of the foundation is mainly examined by four factors, as shown in Table 1. The influence of the upper layer soil on the ultimate bearing capacity of the foundation mainly considers six factors, as shown in Table 3, and each factor considers three levels. Table 2 and Table 4 respectively show the design scheme of the upper and lower layers and the experimental results.

TABLE 1 TABLE OF FACTORS AFFECTING THE UNDERLYING SOIL

Factor level	Influencing factor			
	1 (A)	2 (B)	3 (C)	4 (D)
	Internal friction angle / °	Severe / kN / m^3	Base width / m	Foundation depth / m
One level	35	17.5	2	0.5
One Second level	40	18.5	4	1
Three levels	45	19.5	6	2

TABLE 2 HEADER DESIGN AND EXPERIMENTAL RESULTS

Number	A	B	C	D	Result
	1	2	3	4	$q_u - 2000$ / (kN / m^3)
1	35	17.5	2	0.5	-875
2	35	18.5	4	1	-281
3	35	19.5	6	2	418
4	40	17.5	4	2	2322
5	40	18.5	6	0.5	345
6	40	19.5	2	1	553
7	45	17.5	6	1	3621
8	45	18.5	2	2	6472

	9	45	19.5	4	0.5	1908
I_j		-738	5068	6150	1378	
II_j		3220	6536	3949	3893	
III_j		12001	2879	4384	9212	
$\bar{I}_j$		-246	1689	2050	459	
$\bar{II}_j$		1073	2179	1316	1298	
$\bar{III}_j$		4000	960	1461	3071	
Extreme difference range		4.25	0.73	1.25	2.61	
		10^3				

It is concluded from Table 2 that the foundation width and internal friction angle are the main factors affecting the ultimate bearing capacity of the foundation. The soil gravity and foundation depth are secondary factors.

TABLE 3 UPPER SOIL INFLUENCING FACTORS LEVEL TABLE

Factor level	Influencing factor					
	1A	2B	3C	4D	5E	6F
	Internal friction	Severe	Cohes-	Upper soil	Base width	Founda-tion depth
	angle °	kN / m^3	ion kPa	thickness m	m	m
One level	28	19	21	0.5	3	0.5
One Second level	22	17.5	16	1.0	4	1.0
Three levels	26	16	11	2.0	5	1.5

According to the analysis of the range of the difference, the internal friction angle of the upper layer has the greatest influence on the ultimate bearing capacity of the foundation, followed by the foundation width and the thickness of the upper layer soil, while the influence of the foundation depth and soil cohesion is secondary.

The above analysis shows that the ultimate bearing capacity of the two-layered soil foundation is mainly affected by the internal friction angle and the foundation width of the soil, and the thickness of the upper soil also has a certain influence. The following will mainly analyze the influence of these three factors on the ultimate bearing capacity of the foundation.

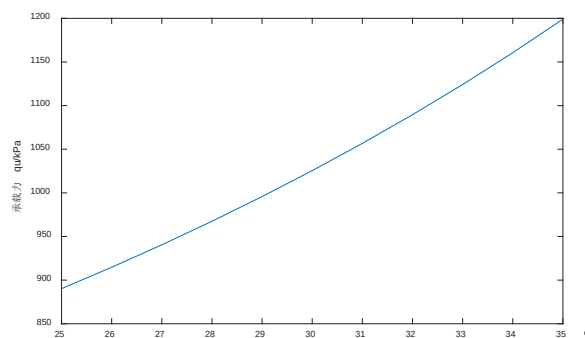


FIGURE 3 EFFECT OF INTERNAL FRICTION ANGLE ON ULTIMATE BEARING CAPACITY OF FOUNDATION

3.2.1 Ultimate Bearing Capacity of Double-Layer Clay Foundation

When the foundation is embedded in a two-layer clay foundation of $\varphi=25^\circ$, it can be divided into two cases: (1) The upper layer of the bearing layer is softer than the lower layer. $c_1 < c_2$ (2) The upper layer of the bearing layer is harder than the lower layer. $c_1 > c_2$

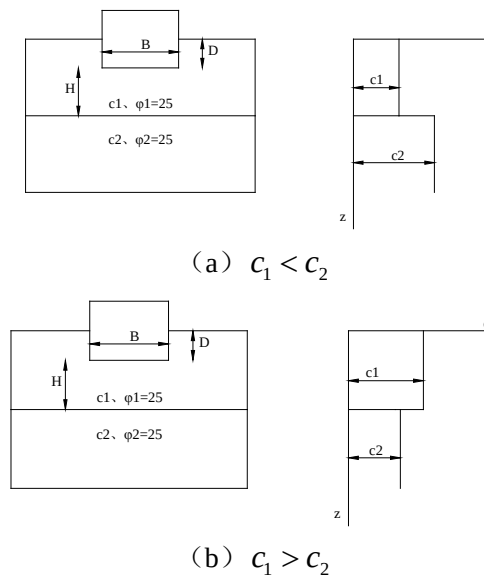


FIGURE 4 $\varphi=25^\circ$ DOUBLE LAYER CLAY FOUNDATION

In the case where the upper layer soil is softer than the lower layer soil, the bearing capacity decreases sharply with the increase of the upper layer soil thickness, and the bearing capacity increases steadily with the increase of the foundation depth. In the case where the upper layer soil is harder than the lower layer soil, the increase in the bearing capacity increases significantly as the thickness of the upper layer increases. As shown in Figure 5.

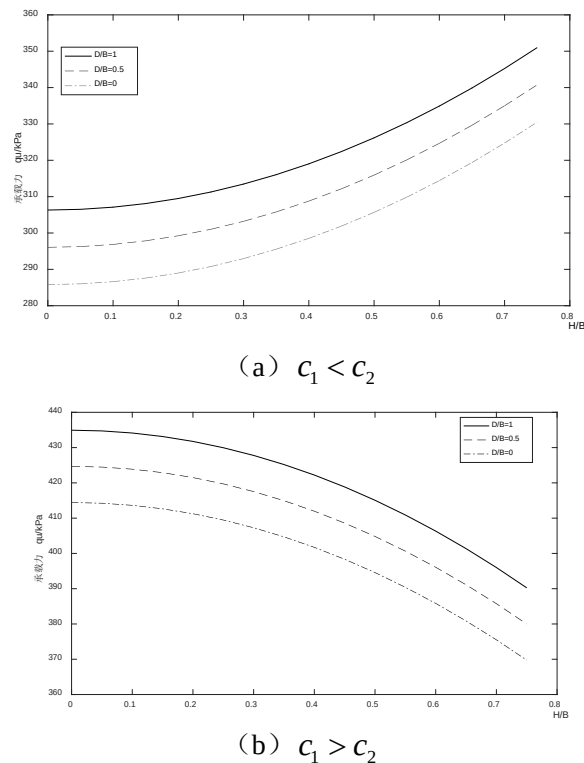


FIGURE 5 TREND CHART OF BEARING CAPACITY WITH THICKNESS-WIDTH RATIO OF UPPER SOIL

3.2.2 Ultimate Bearing Capacity Coefficient of Double-Layer Clay Foundation

As shown in Fig. 6, as the thickness of the upper layer increases, the upper layer soil plays more and more roles, the bearing capacity coefficient N_{c1} increases, and the value of N_{c2} decreases. The thickness of the upper layer does not affect the value of N_q .

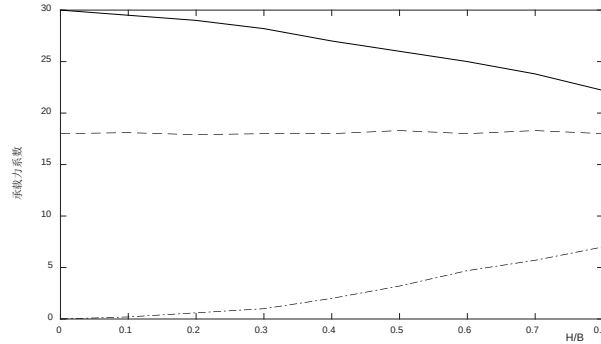


FIGURE 6 BEARING CAPACITY COEFFICIENTS

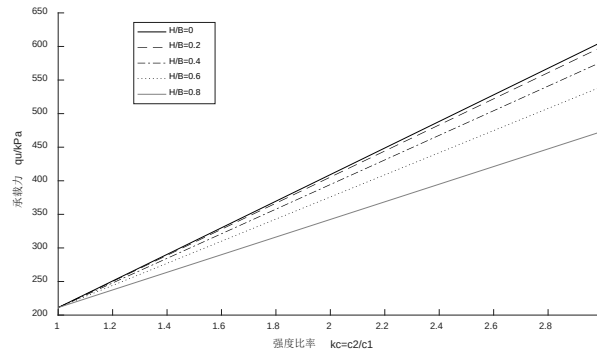


FIGURE 7 BEARING CAPACITY AT DIFFERENT STRENGTH RATIOS

3.2.3 Ultimate Bearing Capacity of Foundation with Weak Underlying Layer

For the calculation of the ultimate bearing capacity of two-layer foundation, the critical depth is a very important parameter. For different critical depths of the same soil layer, the calculated ultimate bearing capacity is very different. The calculation of the two-layer foundation with different strength ratios shows that the upper and lower layers have different strength ratios, the critical depth qu_1 / qu_2 is different, and increases with the increase of the relative strength of the upper layer. (qu_1 、 qu_2 respectively represent the ultimate bearing capacity calculated corresponding to the strength index of the upper and lower layers of soil)

CONCLUSION

Based on the basic concept of Plandell's hypothesis, this paper analyzes the bearing capacity of two-layer foundation by applying the principle of plastic balance, and obtains the formula of bearing capacity of two-layer foundation, and analyzes the change of bearing capacity of foundation under different soil conditions. in conclusion:

- (1) For homogeneous soil, the formula obtained in this paper is the same as the Plandell formula;
- (2) The cohesive force of the soil, the soil bulk density above the foundation, and the aspect ratio have a much smaller influence on the bearing capacity of the foundation than the internal friction angle;
- (3) The upper and lower layers of different strength ratios have different critical depths and increase with the

increase of the relative strength of the upper layers.

(4) This paper assumes that the foundation is completely smooth, and ignores the influence of the weight of the foundation soil, which is not consistent with the sand, but the effect of the friction on both sides of the foundation, the shear strength of the soil and the dilatancy angle on the bearing capacity. It will continue to be explored in subsequent studies.

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