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Adaptive Control Design for High-order MIMO Nonlinear Time-delay Systems Based on Neural Network

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Abstract

An adaptive controller for a class of high-order MIMO nonlinear time-delay systems in block-triangular form is proposed in the paper. The radial basis function neural network is chosen to approximate the unknown nonlinear functions in the system dynamics at first. Lyapunov–Krasovskii functionals are used to compensate the influence of delay terms. Then an adaptive neural network output tracking controller is designed by using the back-stepping recursive method. Based on Lyapunov stability theory, the proposed controller can guarantee all closed-loop signals are globally, uniformly and ultimately bounded, while the output tracking is able to converge to a neighborhood of the origin. Finally, a simulation example is given to illustrate the correctness of the theoretical results.

Keywords: Adaptive Control; High-order Nonlinear Delay Systems; MIMO; Output Tracking; Back-stepping Design; RBF Neural Network

1 INTRODUCTION

In practice, many control systems are multi-input multi-output (MIMO) multivariable systems. As we all known that it is difficult to analyze and control these nonlinear uncertain systems in view of the complex coupling between input and output variables and the time-delay. In [1], the integral Lyapunov functions were used to obtain adaptive fuzzy controller for the first uncertain MIMO nonlinear systems with the block-triangular functions. In [2], a fuzzy adaptive control scheme based on observer was proposed for a class of MIMO nonlinear systems with immeasurable states, and the output feedback control laws and parameter adaptive laws were derived. In [3], hyperbolic tangent functions were used to solve the singularity problem of Lyapunov synthesis in MIMO nonlinear time delay systems.

However, most of the exiting work mainly focuses on the first-order MIMO nonlinear systems. Little attention has been paid to the high-order nonlinear system with time-delay. It is difficult to get the analytical solutions of high-order nonlinear system because of the large inertia. The traditional approaches to study the high-order systems mainly include the fast order reduction of model [5], the approximation of first or second order inertia and pure delay link, sliding mode control [6], fuzzy adaptive control as well as robust adaptive control [7]. In [8], for a class of high-order nonlinear dynamic systems, a composite control strategy was proposed based on adaptive terminal sliding mode control and disturbance observer theory, which achieves sliding mode controller’s switching gain. In [9], for the tracking control for a class of high order nonlinear systems with input delay, the fuzzy logic approach was used to estimate the unknown continuous functions, and a state conversion method was presented to eliminate the delayed input items. Finally, the proposed closed 1oop system was concluded to be semi-globally uniformly ultimately bounded and stable.

In this paper, based on the back-stepping and neural network approximation, we further take into account the tracking control problem for a class of MIMO high-order nonlinear time delay systems with unknown nonlinear functions in block-triangular form. To design an adaptive controller for theses MIMO high-order nonlinear systems, we apply the RBF (Radial Basis Function) neural network to approximate the unknown nonlinear functions, which reduces the difficulty of the controller design. Moreover, we adjust the norm of the neural network weight vector instead of the weight vector. As a result, the adjusting parameters are greatly reduced, and the simulating verification is simplified.

2 PROBLEM FORMULATION AND PRELIMINARIES

Consider the n-input n-output continuous-time MIMO nonlinear system in block-triangular form with unknown time delays described by (1).

$$x^T y \le \frac{\varepsilon^p}{p}\|x\|^p + \frac{1}{q\varepsilon^p}\|y\|^q \tag{1}$$

For $\varepsilon > 0$, $p > 1$, $q > 1$ are the delay state variables of the j subsystem. $(p-1)(q-1)=1$ with $\bar{x}_{j,i_j} = [x_{j,1}, x_{j,2}, \cdots, x_{j,i_j}]^T \in R^{i_j}$ is the vector of delay-free states for the first i_j differential equations of the j subsystem; $\bar{u}_j = [u_1, u_2, \cdots, u_j]^T$ is the inputs for the first j subsystems; $y = [y_1, y_2, \cdots, y_n]^T \in R^n$ is the outputs; τ_{j,i_j} and τ_{j,m_j} are time delays ; $\tau_{j,i_j} > 0$, $\tau_{j,m_j} > 0$; $f_{j,i_j}(\cdot)$, $g_{j,i_j}(\cdot)$ and y_j are unknown smooth functions.

The control objective: (1).Design an adaptive NN feedback controller to ensure closed-loop systems globally bounded; (2).The output y_j follows the specified desired trajectory y_{dj} .

Assumption 1: For $j = 1, 2, \cdots, n$, $i_j = 1, 2, \cdots, m_j - 1$ .The functions $c_{j,i_j}(\cdot)$ and $\bar{c}_{j,i_j}(\cdot)$ satisfy $0 < c_{j,i_j}(\cdot) \le g_{j,i_j}(\cdot) \le \bar{c}_{j,i_j}(\cdot)$, $x_{j,m_j+1} = u_j$.

Assumption 2: Let $p_{j,1}, p_{j,2}, \cdots, p_{j,m_j}$ be odd positive integers.

Assumption 3: The desired trajectories y_{dj} , $j = 1, 2, \cdots, n$, and their time derivatives up to the nth order, are continuous and bounded.Young inequality: for any two vectors x and y ,the following inequality holds: $x^T y \le \frac{\varepsilon^p}{p}\|x\|^p + \frac{1}{q\varepsilon^p}\|y\|^q$ where $\varepsilon > 0$, $p > 1$ and $q > 1$ are constants, such that $(p-1)(q-1) = 1$ .Gromwell inequality: $V_{j,1} = V_{z_{j,1}} + V_{U_{j,1}} + \frac{\tilde{\lambda}_{j,1}^{\ 2}}{2w_{j,1}}$, if the continuous function $u(t)$ satisfies. $u(t) \le M + k\int_{t_0}^{t} u(s)ds$ in $[t_0, t_1]$, then $w_{j,1} > 0$, $\upsilon_{j,1} > 0$, $l_{j,1} > 0$.

Lemma 1: For any real numbers a , b and $p \ge 1$, the following inequality holds: $\lambda_{j,i_j} = \|W_{j,i_j}\|^2$

3 ADAPTIVE NN CONTROLLER DESIGN

In order to reduce the adjusting number of neural network adaptive parameters in the simulation, we define a new unknown constant $\lambda_{j,i_j} = \|W_{j,i_j}\|^2$ .In this paper, we estimate the λ_{j,i_j} instead of the ideal weights of network W_{j,i_j} with Lyapunov method. Thus, we can only adjust one parameter in each system. Define $\hat{\lambda}_{j,i_j}$ is the estimate of λ_{j,i_j} , so the estimate error $\tilde{\lambda}_{i,i_j} = \lambda_{i,i_j} - \hat{\lambda}_{i,i_j}$:

Step 1: $(j,1)$ Let $z_{j,1} = x_{j,1} - y_{dj}$ for $j = 1, 2, \cdots, n$, consider the Lyapunov function as follows: $V_{z_{j,1}} = \frac{z_{j,1}^{\ 2}}{2}$

Then, the time derivative of $V_{z_{j,1}}$ is given by (2)

$$\dot{V}_{z_{j,1}} = z_{j,1}(f_{j,1} + g_{j,1}x_{j,2}^{\ p_{j,1}} + h_{j,1} - \dot{y}_{dj}) \tag{2}$$

With Young inequality, We have $z_{j,1}h_{j,1}(x_{j,1}(t-\tau_{j,1})) \le \frac{1}{2}z_{j,1}^{\ 2} + \frac{1}{2}h_{j,1}^{\ 2}(x_{j,1}(t-\tau_{j,1}))$ Substituting this inequality into (2) yields $\dot{V}_{Z_{j,1}} \le z_{j,1}(f_{j,1} + g_{j,1}x_{j,2}^{\ p_{j,1}} + \frac{1}{2}z_{j,1} - \dot{y}_{dj}) + \frac{1}{2}h_{j,1}^{\ 2}(x_{j,1}(t-\tau_{j,1}))$. To deal with the delay term, consider the

Lyapunov-Krasovskii functional as follows: $V_{U_{j,1}} = \frac{1}{2}\int_{t-\tau_{j,1}}^{t} U_{j,1}{}^{2}(x_{j,1}(s))ds$ Differentiating $V_{U_{j,1}}$ with respect to time, we obtain (3).

$$\dot{V}_{U_{j,1}} = \frac{1}{2}U_{j,1}(x_{j,1}(t)) - \frac{1}{2}U_{j,1}(x_{j,1}(t-\tau_{j,1})). \tag{3}$$

If we choose $U_{j,1}(x_{j,1}(t-\tau_{j,1})) = h_{j,1}{}^{2}(x_{j,1}(t-\tau_{j,1}))$, Then $U_{j,1}(x_{j,1}(t)) = h_{j,1}{}^{2}(x_{j,1}(t))$. Suppose $h_{j,1}{}^{2}(x_{j,1}(t)) \le z_{j,1}{}^{2}\rho_{j,1}{}^{2}(x_{j,1}(t))$, where $\rho_{j,1}{}^{2}(x_{j,1}(t))$ is a known function. Putting (2) and (3) together, we have (4).

$$\dot{V}_{z_{j,1}} + \dot{V}_{U_{j,1}} \le z_{j,1}[F_{j,1}(Z_{j,1}) + g_{j,1}x_{j,2}{}^{p_{j,1}}], \tag{4}$$

where $F_{j,1}(Z_{j,1}) = f_{j,1} + \frac{1}{2}z_{j,1} - \dot{y}_{dj} + +\frac{1}{2}z_{j,1}\rho_{j,1}{}^{2}(x_{j,1}(t))$, $Z_{j,1} = [x_{j,1}, y_{dj}, dy_{dj}]^{T}$. Since the neural networks (NN) have good ability of approximation, we use NN to approximate $F_{j,1}$ such that for given $\varepsilon_{j,1}^{*} > 0$,

$$F_{j,1} = W_{j,1}{}^{T}S(Z_{j,1}) + \varepsilon_{j,1}(Z_{j,1}), \tag{5}$$

where $|\varepsilon_{j,1}| \le \varepsilon_{j,1}^{*}$ is an unknown constant. By Young's inequality and the definition of λ_{j,i_j} , we have (6) (7).

$$z_{j,1}W_{j,1}{}^{T}S(Z_{j,1}) \le \frac{\lambda_{j,1}}{2v_{j,1}{}^{2}}S_{j,1}{}^{T}(Z_{j,1})S(Z_{j,1})z_{j,1}{}^{2} + \frac{v_{j,1}{}^{2}}{2}, \tag{6}$$

$$z_{j,1}\varepsilon_{j,1}(Z_{j,1}) \le \frac{1}{2}z_{j,1}{}^{2} + \frac{1}{2}\varepsilon_{j,1}^{*2}. \tag{7}$$

By utilizing (5)-(7), (4) can be rewritten in the following form.

$$\dot{V}_{z_{j,1}} + \dot{V}_{U_{j,1}} \le \frac{\lambda_{j,1}}{2\upsilon_{j,1}{}^{2}}S_{j,1}{}^{T}(Z_{j,1})S(Z_{j,1})z_{j,1}{}^{2} + \frac{\upsilon_{j,1}{}^{2}}{2} + \frac{1}{2}z_{j,1}{}^{2} + \frac{1}{2}\varepsilon_{j,1}{}^{*2} + z_{j,1}g_{j,1}x_{j,2}{}^{p_{j,1}}. \tag{8}$$

We further consider the Lyapunov function as follows: $V_{j,1} = V_{z_{j,1}} + V_{U_{j,1}} + \frac{\tilde{\lambda}_{j,1}{}^{2}}{2w_{j,1}}$. Design NN adaptive law as follows:

$$\dot{\hat{\lambda}}_{j,1} = \frac{w_{j,1}}{2\upsilon_{j,1}{}^{2}}S^{T}(Z_{j,1})S(Z_{j,1})z_{j,1}{}^{2} - l_{j,1}\hat{\lambda}_{j,1}, \tag{9}$$

where $w_{j,1} > 0$, $\upsilon_{j,1} > 0$, $l_{j,1} > 0$ are constants to be selected. From (8) and (9), we have

$$\dot{V}_{j,1} \le \frac{\hat{\lambda}_{j,1}}{2\upsilon_{j,1}{}^{2}}S^{T}(Z_{j,1})S(Z_{j,1})z_{j,1}{}^{2} + \frac{\upsilon_{j,1}{}^{2}}{2} + \frac{1}{2}z_{j,1}{}^{2} + \frac{1}{2}\varepsilon_{j,1}{}^{*2} + z_{j,1}g_{j,1}x_{j,2}{}^{p_{j,1}} + \frac{1}{w_{j,1}}\tilde{\lambda}_{j,1}l_{j,1}\hat{\lambda}_{j,1}$$

As we know

$$\frac{l_{j,1}}{w_{j,1}}\tilde{\lambda}_{j,1}\hat{\lambda}_{j,1} \le -\frac{l_{j,1}}{2w_{j,1}}\tilde{\lambda}_{j,1}{}^{2} + \frac{l_{j,1}}{2w_{j,1}}\lambda_{j,1}{}^{2}. \tag{10}$$

Design virtual smooth controller:

$$\alpha_{j,2} = -\left(z_{j,1}\frac{n + \frac{1}{2} + \frac{\hat{\lambda}_{j,1}}{2\upsilon_{j,1}{}^{2}}S^{T}(Z_{j,1})S(Z_{j,1})}{c_{j,1}(x_{j,1})}\right)^{\frac{1}{p_{j,1}}}. \tag{11}$$

By assumption 1 and $-z_{j,1}{}^{p_j - p_{j,1}+1}\alpha_{j,2}{}^{p_{j,1}} \ge 0$, we arrive at

$$-c_{j,1}z_{j,1}{}^{p_j - p_{j,1}+1}\alpha_{j,2}{}^{p_{j,1}} \le -g_{j,1}z_{j,1}{}^{p_j - p_{j,1}+1}x_{j,2}{}^{p_{j,1}} \tag{12}$$

From (10)-(12), we have $\dot{V}_{j,1} \le -nz_{j,1}{}^{2} - \frac{l_{j,1}}{2w_{j,1}}\tilde{\lambda}_{j,1}{}^{2} + \bar{c}_{j,1}\left|z_{j,1}\right|\left|x_{j,2}{}^{p_{j,1}} - \alpha_{j,2}{}^{p_{j,1}}\right| + \frac{\upsilon_{j,1}{}^{2}}{2} + \frac{1}{2}\varepsilon_{j,1}{}^{*2} + \frac{l_{j,1}}{2w_{j,1}}\lambda_{j,1}{}^{2}$.

Step 2 : (j, i_j ($j = 1, 2, \cdots, n$, $i_j = 2, \cdots, m_j - 1$)).Let $z_{j,i_j} = x_{j,i_j} - \alpha_{j,i_j}$, construct the Lyapunov function as

$V_{z_{j,i_j}} = \frac{z_{j,i_j}{}^2}{2}$. A direct calculation gives $\dot{V}_{z_{j,i_j}} = z_{j,i_j}(f_{j,i_j} + g_{j,i_j} x_{j,i_j+1}{}^{p_{j,i_j}} + h_{j,i_j} - \dot{\alpha}_{j,i_j})$. Similar to step 1, we have $z_{j,i_j} h_{j,i_j}(x_{j,i_j}(t-\tau_{j,i_j})) \le \frac{1}{2} z_{j,i_j}{}^2 + \frac{1}{2} h_{j,i_j}{}^2(x_{j,i_j}(t-\tau_{j,i_j}))$. Note that $\dot{\alpha}_{j,i_j}$ can be expressed as

$$\dot{\alpha}_{j,i_j} = \sum_{k=1}^{i_j} \frac{\partial \alpha_{j,i_j}}{\partial x_{j,k}} (f_{j,k} + g_{j,k} x_{j,k+1}{}^{p_{j,k}}) + \sum_{k=1}^{i_j} \frac{\partial \alpha_{j,i_j}}{\partial x_{j,k}} h_{j,k}(x_{\tau_{j,k}}) + \sum_{k=0}^{i_j} \frac{\partial \alpha_{j,i_j}}{\partial y_{dj}{}^{(k)}} y_{dj}{}^{(k+1)} + \sum_{k=1}^{i_j} \frac{\partial \alpha_{j,k}}{\partial \hat{\lambda}_{j,k}} \dot{\hat{\lambda}}_{j,k}.$$

We have:

$$\begin{aligned} \dot{V}_{z_{j,i_j}} &\le z_{j,i_j}[f_{j,i_j} + g_{j,i_j} x_{j,i_j+1}{}^{p_{j,i_j}} + \frac{1}{2} z_{j,i_j} - \sum_{k=1}^{i_j} \frac{\partial \alpha_{j,i_j}}{\partial x_{j,k}} (f_{j,k} + g_{j,k} x_{j,k+1}{}^{p_{j,k}}) - \sum_{k=0}^{i_j} \frac{\partial \alpha_{j,i_j}}{\partial y_{dj}{}^{(k)}} y_{dj}{}^{(k+1)} \\ &\quad - \sum_{k=1}^{i_j} \frac{\partial \alpha_{j,k}}{\partial \hat{\lambda}_{j,k}} \dot{\hat{\lambda}}_{j,k} + \frac{1}{2} \sum_{k=1}^{i_j} z_{j,i_j} (\frac{\partial \alpha_{j,i_j}}{\partial x_{j,k}})^2] + \frac{1}{2} h_{j,i_j}{}^2(x_{\tau_{j,i_j}}) + \frac{1}{2} \sum_{k=1}^{i_j} h_{j,k}{}^2(x_{\tau_{j,k}}). \end{aligned} \tag{13}$$

To deal with the delay term, consider the Lyapunov-Krasovskii functional as follows:

$V_{U_{j,i_j}} = \int_{t-\tau_{j,i_j}}^{t} \frac{1}{2} \mu_{j,i_j}(x(s)) ds + \frac{1}{2} \sum_{k=1}^{i_j-1} \int_{t-\tau_{j,k}}^{t} \mu_{j,k}(x(s)) ds.$ Differentiating $V_{U_{j,i_j}}$ yields

$$\dot{V}_{U_{j,i_j}} = \frac{1}{2} \mu_{j,i_j}(x(t)) - \frac{1}{2} \mu_{j,i_j}(x(t-\tau_{j,i_j})) + \frac{1}{2} \sum_{k=1}^{i_j-1} \mu_{j,k}(x(t)) - \frac{1}{2} \sum_{k=1}^{i_j-1} \mu_{j,k}(x(t-\tau_{j,k})). \tag{14}$$

If we choose $\mu_{j,i_j}(x(t-\tau_{j,i_j})) + \sum_{k=1}^{i_j} \mu_{j,k}(x(t-\tau_{j,k})) = h_{j,i_j}{}^2(x(t-\tau_{j,i_j})) + \sum_{k=1}^{i_j} h_{j,k}{}^2(x(t-\tau_{j,k}))$,

then $\mu_{j,i_j}(x(t)) + \sum_{k=1}^{i_j} \mu_{j,k}(x(t)) = h_{j,i_j}{}^2(x(t)) + \sum_{k=1}^{i_j} h_{j,k}{}^2(x(t))$. Suppose:

$$h_{j,i_j}{}^2(x(t)) + \sum_{k=1}^{i_j} h_{j,k}{}^2(x(t)) \le z_{j,i_j}{}^2(\rho_{j,i_j}{}^2(x(t)) + \sum_{k=1}^{i_j} \rho_{j,k}{}^2(x(t))), \tag{15}$$

where $\rho_{j,i_j}(x(t))$, $\rho_{j,k}(x(t))$ are known functions. From (13)-(15), we can get

$$\dot{V}_{z_{j,i_j}} + \dot{V}_{U_{j,i_j}} \le z_{j,i_j}(F_{j,i_j}(Z_{j,i_j}) + g_{j,i_j} x_{j,i_j+1}{}^{p_{j,i_j}}),$$

where:

$$\begin{aligned} F_{j,i_j} &= f_{j,i_j} + \frac{1}{2} z_{j,i_j}{}^{p_j - p_{j,i_j}+1} - \sum_{k=0}^{i_j} \frac{\partial \alpha_{j,i_j}}{\partial y_{dj}{}^{(k)}} y_{dj}{}^{(k+1)} - \sum_{k=1}^{i_j} \frac{\partial \alpha_{j,i_j}}{\partial x_{j.k}} (f_{j,k} + g_{j,k} x_{j,k+1}{}^{p_{j,k}}) - \sum_{k=1}^{i_j-1} \frac{\partial \alpha_{j,i_j}}{\partial \hat{\lambda}_{j,k}} \dot{\hat{\lambda}}_{j,k} \\ &\quad + \sum_{k=1}^{i_j} \frac{1}{2} z_{j,i_j}{}^{p_j - p_{j,i_j}+1} (\frac{\partial \alpha_{j,i_j}}{\partial x_{j,k}})^2 + \frac{1}{2} z_{j,i_j}(\rho_{j,i_j}{}^2(x(t)) + \sum_{k=1}^{i_j} \rho_{j,k}{}^2(x(t))). \end{aligned}$$

Similar to step 1, we can get $F_{j,i_j} = W_{j,i_j}{}^T S(Z_{j,i_j}) + \varepsilon_{j,i_j}(Z_{j,i_j})$,

$$\dot{V}_{z_{j,i_j}} + \dot{V}_{U_{j,i_j}} \le \frac{\lambda_{j,i_j}}{2\upsilon_{j,i_j}{}^2} S^T(Z_{j,i_j}) S(Z_{j,i_j}) z_{j,i_j}{}^2 + \frac{1}{2} \upsilon_{j,i_j}{}^2 + \frac{1}{2} \varepsilon_{j,i_j}{}^{*2} + \frac{1}{2} z_{j,i_j}{}^2 + z_{j,i_j} g_{j,i_j} x_{j,i_j+1}{}^{p_{j,i_j}},$$

where $Z_{j,i_j} = [\bar{x}_{j,i_j}{}^T, y_{dj}, \cdots, y_{dj}{}^{i_j}, \lambda_{j,1}, \cdots, \hat{\lambda}_{j,i_j-1}]$. Consider the Lyapunov function as follows:

$$V_{j,i_j} = V_{j,i_j-1} + V_{z_{j,i_j}} + V_{U_{j,i_j}} + \frac{\tilde{\lambda}_{j,i_j}{}^2}{2w_{j,i_j}}$$

Since

$$\begin{aligned} \dot{V}_{j,i_j-1} &\le -(n - i_j + 2)(z_{j,1}{}^2 + \cdots + z_{j,i_j-1}{}^2) - \sum_{k=1}^{i_j-1} \frac{l_{j,k}}{2w_{j,k}} \tilde{\lambda}_{j,k}{}^2 \\ &\quad + \sum_{k=1}^{i_j-1} (\frac{\upsilon_{j,k}{}^2}{2} + \frac{1}{2} \varepsilon_{j,k}{}^{*2} + \frac{l_{j,k}}{2w_{j,k}} \lambda_{j,k}{}^2) + \bar{c}_{j,i_j-1} \left| z_{j,i_j-1} \right| \left| x_{j,i_j}{}^{p_{j,i_j-1}} - \alpha_{j,i_j}{}^{p_{j,i_j-1}} \right| \end{aligned}$$

Young inequality and Lemma 1 imply the existence of a smooth function $\tilde{\beta}_{j,i_j}(\cdot) > 0$, such that.

$\bar{c}_{j,i_j-1}\left|z_{j,i_j-1}\right|\left|x_{j,i_j}{}^{p_{j,i_{j-1}}} - \alpha_{j,i_j}{}^{p_{j,i_{j-1}}}\right| \le \sum_{l=1}^{i_j-1} z_{j,l}{}^2 + z_{j,i_j}{}^2 \tilde{\beta}_{j,i_j}(\cdot)$ Then a virtual smooth controller of the form

$\alpha_{j,i_j+1} = -(z_{j,i_j}\dfrac{n - i_j + 1 + \dfrac{\hat{\lambda}_{j,i_j}}{2\upsilon_{j,i_j}{}^2} S^T(Z_{j,i_j})S(Z_{j,i_j}) + \dfrac{1}{2} + \tilde{\beta}_{j,i_j}}{c_{j,i_j}})^{\frac{1}{p_{j,i_j}}}$ is such that:

$$\dot{V}_{j,i_j} \le -(n - i_j + 1)\sum_{k=1}^{i_j} z_{j,k}{}^2 - \sum_{k=1}^{i_j} \frac{l_{j,k}}{2w_{j,k}} \tilde{\lambda}_{j,k}^2 + \sum_{k=1}^{i_j} (\frac{\upsilon_{j,k}{}^2}{2} + \frac{1}{2}\varepsilon_{j,k}{}^{*2} + \frac{l_{j,k}}{2w_{j,k}} \lambda_{j,k}{}^2) + \bar{c}_{j,i_j} z_{j,i_j}\left|x_{j,i_j+1}{}^{p_{j,ij}} - \alpha_{j,i_j+1}{}^{p_{j,ij}}\right|$$

Step $j, m_j(j = 1,\cdots,n)$ *:* let $z_{j,m_j} = x_{j,m_j} - \alpha_{j,m_j}$, construct the Lyapunov function $V_{z_{j,m_j}} = \frac{z_{j,m_j}{}^2}{2}$ A straightforward calculation gives $\dot{V}_{z_{j,m_j}} = z_{j,m_j}(f_{j,m_j} + g_{j,m_j} u_j{}^{p_{j,m_j}} + h_{j,m_j} - \dot{\alpha}_{j,m_j})$.To deal with the delay term, consider the Lyapunov-Krasovskii functional as follows: $V_{U_{j,m_j}} = \int_{t-\tau_{j,m_j}}^{t} \frac{1}{2}\mu(x(s))ds + \sum_{k=1}^{m_j-1}\int_{t-\tau_{j,k}}^{t} \frac{1}{2}\mu(x(s))ds$ Design NN adaptive law as follows: $\dot{\hat{\lambda}}_{j,m_j} = \frac{w_{j,m_j}}{2\upsilon_{j,m_j}{}^2} S^T(Z_{j,m_j})S(Z_{j,m_j}) z_{j,m_j}{}^2 - l_{j,m_j}\hat{\lambda}_{j,m_j}$ Similar to step 2, choose Lyapunov function as follows

$$V_{j,m_j} = V_{j-1,m_{(j-1)}} + V_{j,m_j-1} + V_{z_{j,m_j}} + V_{U_{j,m_j}} + \frac{\tilde{\lambda}_{j,m_j}^2}{2w_{j,m_j}} \tag{16}$$

Then, the time derivative of V_{j,m_j} is given by

$$\dot{V}_{j,m_j} \le -(n - m_j + 1)\sum_{l=1}^{m_j} z_{j,l}{}^2 - \sum_{i=1}^{j}\sum_{k=1}^{m_j} \frac{l_{i,m_j}}{2w_{i,i_j}} \tilde{\lambda}_{i,k}^2 + \sum_{i=1}^{j}\sum_{k=1}^{m_j} (\frac{\upsilon_{i,k}{}^2}{2} + \frac{1}{2}\varepsilon_{i,k}{}^{*2}$$
$$+ \frac{l_{i,k}}{2w_{i,k}} \lambda_{i,k}{}^2) + \bar{c}_{j,m_j} z_{j,m_j}\left|x_{j,m_j+1}{}^{p_{j,m_j}} - \alpha_{j,m_j+1}{}^{p_{j,m_j}}\right|$$

After step n, m_n *:* the last step for the nth subsystem, it can be shown that the derivative of n, m_n along the closed-loop trajectories satisfies the following inequality:

$$\begin{aligned}\dot{V}_{n,m_n} \le & -\sum_{j=1}^{n}\sum_{k=1}^{m_j} (z_{j,k}{}^2 + \frac{l_{j,k}}{2w_{j,k}} \tilde{\lambda}_{j,k}^2) + \sum_{j=1}^{n}\sum_{k=1}^{m_j} (\frac{\upsilon_{j,k}{}^2}{2} + \frac{1}{2}\varepsilon_{j,k}{}^{*2} + \frac{l_{j,k}}{2w_{j,k}} \lambda_{j,k}{}^2) \\ & + c_{n,m_n} z_{n,m_n}\left|x_{n,m_n+1}{}^{p_{n,m_n}} - \alpha_{n,m_n+1}{}^{p_{n,m_n}}\right|\end{aligned} \tag{17}$$

From assumption 1, we know $x_{n,m_n+1} = u_n$. Combined with the formula (12), we can obtain adaptive designed controller and adaptation law are as follows

$$u_n = -(z_{n,m_n}\frac{1 + \dfrac{\hat{\lambda}_{n,m_n}}{2\upsilon_{n,m_n}{}^2} S^T(Z_{n,m_n})S(Z_{n,m_n}) + \dfrac{1}{2} + \tilde{\beta}_{n,m_n}}{c_{n,m_n}})^{\frac{1}{p_{n,m_n}}}, \tag{18}$$

$$\dot{\hat{\lambda}}_{n,m_n} = \frac{w_{n,m_n}}{2\upsilon_{n,m_n}{}^2} S^T(Z_{n,m_n})S(Z_{n,m_n}) z_{n,m_n}{}^2 - l_{n,m_n}\hat{\lambda}_{n,m_n}, \tag{19}$$

such that

$$\dot{V}_{n,m_n} \le -\sum_{j=1}^{n}\sum_{k=1}^{m_j} (\frac{l_{j,k}}{2w_{j,k}} \tilde{\lambda}_{j,k}^2 + z_{j,k}{}^2) + C, \tag{20}$$

where $C = \sum_{j=1}^{n}\sum_{k=1}^{m_j} (\frac{\upsilon_{j,k}{}^2}{2} + \delta_{j,k} + \frac{l_{j,k}}{2w_{j,k}} \lambda_{j,k}{}^2)$ is a constant. Theorem 1 Closed-loop systems (1) that satisfy the assumptions 1-4, there exists a state feedback controller u such as (18) and NN adaptation law $\dot{\hat{\lambda}}$ such as (19), such that all the closed-loop trajectories remain bounded and the output tracking error converges to a neighbourhood of the origin. Proof. From (19), we have $\dot{V}_{n.m_n} \le -cV_{n,m_n} + \psi$, where $c = \min\{2, l_{j,k}\}$,

$$\psi = \sum_{j=1}^{n}\sum_{k=1}^{m_j}(\frac{\upsilon_{j,k}^{2}}{2} + \delta_{j,k} + \frac{l_{j,k}}{2w_{j,k}}\lambda_{j,k}^{2}) + \int_{t-\tau_{j,i_j}}^{t}\frac{1}{2}\mu_{j,i_j}(x(s))ds + \frac{1}{2}\sum_{k=1}^{i_j-1}\int_{t-\tau_{j,k}}^{t}\mu_{j,k}(x(s))ds.$$

By Gromwell inequality, we can obtain:

$$V_{n,m_n} \le \frac{\psi}{c} + \exp^{-ct}[V_{n,m_n}(z(0)) - \frac{\psi}{c}] \le V_{n,m_n}(z(0)) + \frac{\psi}{c}. \tag{21}$$

Since $V_{n,m_n}(z(0))$ is bounded, then z_{j,i_j} , x_{j,i_j} and $\hat{\lambda}_{j,i_j}$ are bounded. From (16) and (21), we have $\frac{z_{n,m_n}^{2}}{2} \le V_{n,m_n} \le V_{n,m_n}(z(0)) + \frac{\psi}{c}$, then $|z_{j,1}| = |x_{j,1} - y_{dj}| \le \sqrt{2(V_{n,m_n}(z(0)) + \frac{\psi}{c})}$. So, the output tracking error converges to a neighborhood of the origin. The proof is finished.

4 SIMULATIONS

In this section, a simulation study is presented to verify the effectiveness of the adaptive NN controller for the general case. In this case, the bounds on the functions of delayed states are not known. Consider the following two-input, two-output system in block-triangular form

$$\begin{cases} \dot{x}_{11} = x_{11} + (1+\sin^2(x_{11}))x_{12}^3 + x_{11}(t-\tau_{11}) \\ \dot{x}_{12} = x_{11}x_{12} + x_{12} + x_{22} + (1+\sin^2(x_{11}) + \cos^2(x_{12}))u_1^5 + x_{12}(t-\tau_{12}) \\ \dot{x}_{21} = -x_{21} + x_{22}^3 + x_{21}(t-\tau_{21}) \\ \dot{x}_{22} = (x_{12}+x_{21})x_{22} - x_{11}u_1 + (2-\sin(x_{21}x_{22} - x_{11})u_2^5 + x_{22}(t-\tau_{22}) \\ y_1 = x_{11} \\ y_2 = x_{21} \end{cases}$$

The desired trajectories $y_{d1} = \cos(t)$, $y_{d2} = \sin(t)$. The simulation results shown in from Fig.1 to Fig.6 are based on the following parameters.

$$w_{11} = w_{12} = w_{21} = w_{22} = 8;\, l_{11} = l_{12} = l_{21} = l_{22} = 0.0001;\, \upsilon_{11} = \upsilon_{12} = 8;\, \upsilon_{21} = \upsilon_{22} = 5;\, c_{11} = \frac{1}{42};\, c_{12} = \frac{1}{5};\, c_{21} = \frac{1}{8};$$

$$c_{22} = \frac{1}{7};[\lambda_{11},\lambda_{12},\lambda_{21},\lambda_{22}]^T = [0,0,0,0]^T,\, \tau_{12} = 0.5,\, \tau_{21} = 1.5,\, \tau_{22} = 0.5.$$

The initial conditions: $[\lambda_{11},\lambda_{12},\lambda_{21},\lambda_{22}]^T = [0,0,0,0]^T$, $[x_{11}(0), x_{12}(0), x_{21}(0), x_{22}(0)]^T = [0,0,0,0]^T$. The NN simulation results clearly show that the proposed controller can guarantee the bounded of all the signals in the closed-loop system From Fig.1 and Fig.2, we can get that the output signal y_1 and $y_1(t)$ can effectively follow the reference $y_{d1}(t)$ and $y_2(t)$ respectively. At the same time, Fig.3 and Fig.4 clearly show the sate signal $y_{d2}(t)$ and x_{21} are globally bounded. From Fig.5 and Fig.6, we can get the tracking error converges to a relatively small neighborhood.

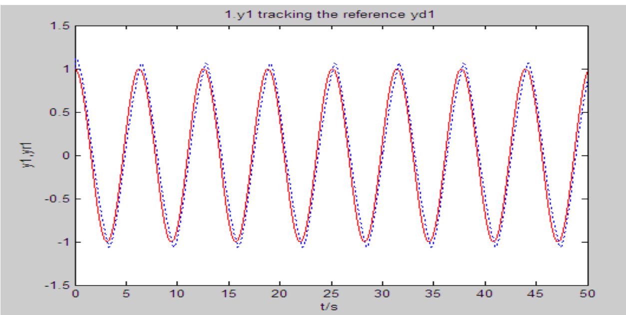

FIG.1. OUTPUT ("- -") AND REFERENCE SIGNAL ("–")

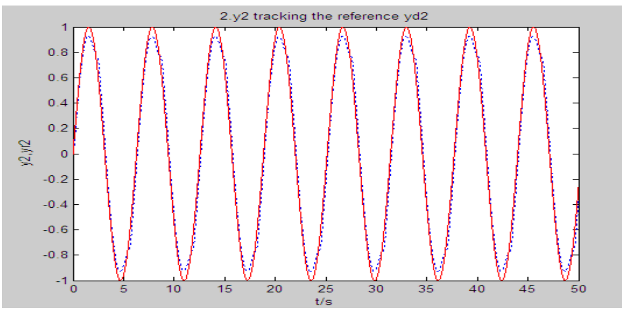

FIG.2 OUTPUT("- -") AND REFERENCE SIGNAL("-")

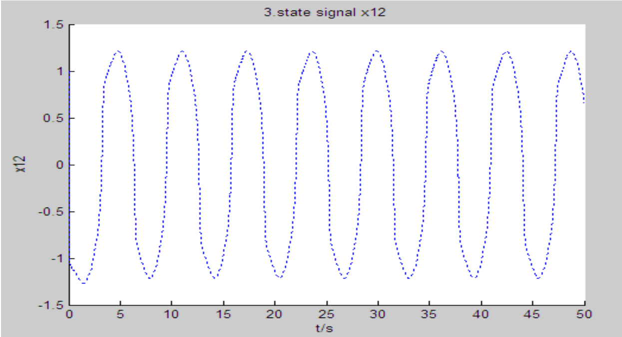

FIG.3 STATE SIGNAL x_{12}

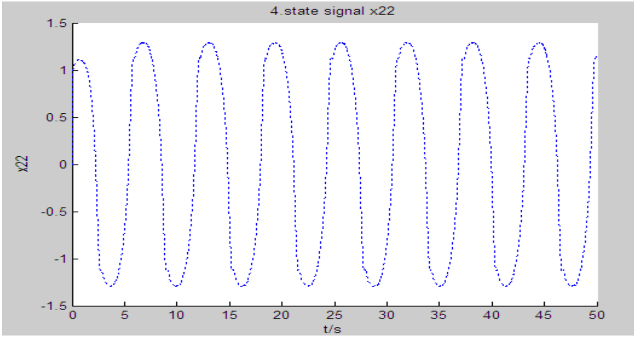

FIG.4.STATE SIGNAL x_{22}

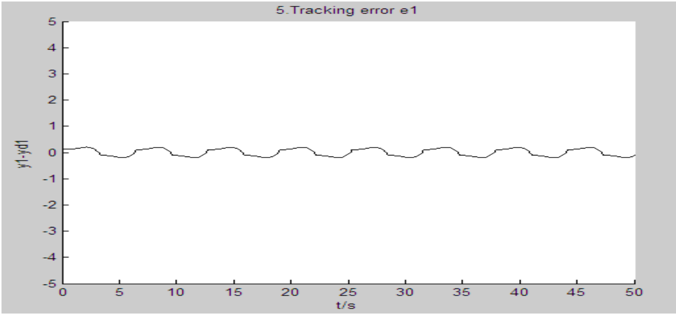

FIG.5. TRACKING ERROR $y_1(t)-y_{d1}(t)$

5 CONCLUSIONS

In this paper, we design the NN controller and adaptive laws for a class of MIMO high-order nonlinear time delay systems, by using back-stepping and the neural network approximation theory. Based on Lyapunov function, the designed adaptive controller can ensure that all the signals of the closed-loop system remain bounded. Moreover, the tracking errors are able to converge to a neighborhood of the origin. Simulation results further show the proposed adaptive controller scheme is feasible.

The MIMO high-order nonlinear systems with variable time delay are more common in the control fields and they are much more complex. Yet we only discuss the situation with constant time delay in the paper. For the future work, we plan to study these systems with variable time delay.

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