Scientific Journal of Control Engineering
February 2014, Volume 4, Issue 1, PP.1-7

Dichotomy and Absolute Stability Analysis of a Class of Complex Dynamical Networks

Haijing Liu, Dun Ao [#] , Lili Xia, Di Liang

College of Electronic Information and Control Engineering, Beijing University of Technology, Beijing 100124, China

[#]Email: aodun@bjut.edu.cn

Abstract

This paper deals with dichotomy and absolute stability problem for a class of complex network systems with each node be a general Lur'e system which has sector constraint functions. The interconnections of complex network systems referred in this paper conclude not only the linear interconnection of states, but also the interconnection of the sector nonlinearities. Besides, bilinear matrix inequality (BMI) conditions of dichotomy and absolute stability of complex network systems are derived. Moreover, controller design methods of dichotomy and absolute stability of complex system are given in this paper and the related controller can be constructed via feasible solutions of a certain set of bilinear matrix inequalities (BMIs). Finally, examples are given to illustrate the effectiveness of the proposed methods.

Keywords: Complex Network System; Lur'e System; Property of Dichotomy; Absolute Stability; BMI (Bilinear Matrix Inequality)

1 INTRODUCTION

Complex dynamic networks have been extensively studied in the recent years by physicists, biologists, social scientists and control scientists [1]. For dynamic network systems, stability criterion of large-scale system is given by Moylan and Hill [2], and the formation problem of airplane using a distributed control method is derived by Wolfe [3]. With the concept of the S-hull, a multi-variable stability criterion similar is given by Lestas and Vinnicombe [4]. The control problems for a spatially invariant system are studied in [5]. The systems considered above are linear systems, while the nonlinearity can't be avoided in practical systems.

Lur'e system is one of the classic nonlinear systems, which can be regarded as a feedback connection of linear system and nonlinear element satisfying some sector conditions [6], which can portray a lot of nonlinear dynamical systems in practical engineering. Nowadays, the study of global properties and robust stability of nonlinear network systems have attracted a lot of attentions of researchers[7-10]. In [7], the stability of distributed heterogeneous systems with static nonlinear interconnections is considered. Considering the interconnection matrix of the network systems, the synchronization conditions [8] and stability conditions [9] are given. The consensus region of multi-agent systems with nonlinear dynamics is investigated in [10]. Reference [11] presents the formation controller for the second-order multiagent systems with time-varying delay and nonlinear dynamics. But the interconnection considered above is only one kind of interconnections.

In this paper, we consider the dichotomy and absolute stability of a class of nonlinear complex dynamical networks with two kinds of interconnections. The complex network system is composed of subsystems, which are a kind of nonlinear system, i.e., Lur'e systems. Besides, the interconnections of the complex network systems referred in this paper conclude not only the linear interconnection of states, but also the interconnection of the sector nonlinearities.

2 DESCRIPTION OF THE PROBLEM

Suppose that every node in a complex network is a Lur'e systems described by

[#] This work was supported by National Science Foundation of China under Grants 61004012.

$$\begin{aligned} \dot{x}_i &= Ax_i + Bu \\ y_i &= Cx_i \qquad\qquad i = 1,\dots,N \\ u_i &= -f_i(y_i) \end{aligned} \tag{1}$$

where $A \in R^{n\times n}, B \in R^{n\times m}, C \in R^{m\times n}, x_i = (x_{i1},\dots,x_{in})^T, y_i = (y_{i1},\dots,y_{im})^T$, $f_i(y_i) = (f_{i1}(y_{i1}),\dots,f_{im}(y_{im}))^T$, and the nonlinear function f_i satisfying $\gamma_i\tau^2 \le f_i(\tau)\tau \le \delta_i\tau^2, \gamma_i \le \delta_i$.

The corresponding transfer function is defined as $K(s) = C(sI - A)^{-1}B$.

Consider a class of dynamic complex network with each node being a general Lur'e system

$$\begin{aligned} \dot{x}_i &= Ax_i + \sum_{j=1, j\ne i}^{N} a_{ij}A_{12}(x_j - x_i) + Bu_i + \sum_{j=1, j\ne i}^{N} a_{ij}B_{12}(u_j - u_i) \\ y_i &= Cx_i \\ u_i &= -f_i(y_i) \end{aligned} \tag{2}$$

Where $x_i \in R^n, y_i \in R^m, A, B, C$ have the same meaning as those in (1). $A_{12} \in R^{n\times n}$, $B_{12} \in R^{n\times n}$ are the inner coupling matrix describing the interconnection among components of $(x_j - x_i)$ and $(u_j - u_i)$, $i, j = 1,\dots,N$ respectively. And $L = (a_{ij})_{N\times N}$ denotes the connection topology and coupling strength $a_{ij}, i \ne j$ are given as in (2) and $a_{ii} = -\sum_{j=1, j\ne i}^{N} a_{ij}$ which are referred to the outer coupling matrix. By the Kronecker product, network (2) can be rewritten as

$$\begin{aligned} \dot{x} &= (I_N \otimes A + L \otimes A_{12})x + (I_N \otimes B + L \otimes B_{12})u \\ y &= (I_N \otimes C)x \\ u &= -f(y) \end{aligned} \tag{3}$$

Lemma 1[6] Suppose that A has no eigenvalues on the imaginary axis and (A, B) is controllable and (A, C) is observable. And suppose that nonlinear function f_i $(i = 1,\cdots,m)$ is piecewise continuously differentiable and $f_i'(\tau)$ is bounded. If system (1) has isolated equilibriums and there exist diagonal matrices *P* and *Q* with *Q*>0, a scalar $\varepsilon > 0$ and a symmetric matrix W such that the following LMI is feasible

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} \\ * & \Pi_{22} \end{bmatrix} \le 0 \tag{4}$$

Where $\Gamma = diag(\gamma_1,\cdots,\gamma_m), \Delta = diag(\delta_1,\cdots,\delta_m)$, $\Pi_{11} = WA + A^TW - C^T\Gamma\Delta QC + \varepsilon A^TC^TCA$,
$\Pi_{22} = PCB + B^TC^TP - Q + \varepsilon B^TC^TCB$,
$\Pi_{12} = WB + A^TC^TP + \varepsilon A^TC^TCB - \frac{1}{2}C^TQ(\Gamma + \Delta)$, then system (1) is dichotomous for all $f \in [\Gamma, \Delta]$.

Lemma 2[12] $T_1 = T_1^T, T_2, T_3$ are compatible dimension matrix. The following conditions are equivalent

(1) $T_1 + He(T_2\Delta T_3) < 0, \forall\Delta : \Delta^T\Delta \le \lambda^2 I$; (2) there exists $\eta > 0$ satisfying $T_1 + \eta\lambda^2 T_2T_2^T + \frac{1}{\eta}T_3^TT_3 < 0$;

(3) there exists $\eta > 0$ satisfying $\begin{bmatrix} T_1 + \eta\lambda^2 T_2T_2^T & T_3^T \\ T_3 & -\eta I \end{bmatrix} < 0$;(4) there exists $\eta > 0$ satisfying $\begin{bmatrix} T_1 + \eta\lambda^2 T_3^TT_3 & T_2 \\ * & -\eta I \end{bmatrix} < 0$.

Lemma 3[9]Suppose λ_i are distinct eignvalues of *L*, then $I_N \otimes A + L \otimes A_{12}$ is stable if and only if $A + \lambda_i A_{12}$ (*i=1,...,N*) are stable simultaneously.

Proof Let *T* be a non-singular matrix such that $T^{-1}LT = J$, where *J* is the diagonal form of *L*. Then the similarity transformation, i.e., $(T^{-1} \otimes I)(I_N \otimes A + L \otimes A_{12})(T \otimes I) = I_N \otimes A + J \otimes A_{12}$ completes the proof .

Lemma 4[6] System (1) is absolutely stable for all $f \in [\Gamma \quad \Delta]$, if $A - B\Gamma C$ is stable and there exist diagonal matrices *P* and *Q* with $Q \ge 0$, and a symmetric matrix *W* such that the following LMI is feasible

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} \\ * & PCB + B^T C^T P - Q \end{bmatrix} < 0 \qquad (5)$$

Where $\Xi_{11} = WA + A^T W - C^T \Gamma \Delta Q C, \Xi_{12} = WB + A^T C^T P - \frac{1}{2} C^T Q(\Gamma + \Delta)$.

3 MAIN RESULTS

Firstly, BMI criterion about dichotomy of nonlinear network system is given. Then the problem of the absolute stability of nonlinear network systems is considered, and the method of designing the related controllers is obtained.

Theorem 1 Suppose L is a matrix with eigenvalues $\lambda_i, i = 1,...N,$ different from each other. If $A_{\lambda_i} = A + \lambda_i A_{12}$, $i = 1,...N$ have no eigenvalues on the imaginary axis. $(A_{\lambda_i}, B_{\lambda_i})$ is controllable, and (A_{λ_i}, C) (A_{λ_i}, C) is observable. If system (3) has isolated equilibrium and there exist diagonal matrices P and Q with Q>0, a scalar $\varepsilon > 0$ and a symmetric matrix W such that the following LMI is feasible

$$\begin{bmatrix} \tilde{\Xi}_{11} & \tilde{\Xi}_{12} \\ * & \tilde{\Xi}_{22} \end{bmatrix} \le 0 \qquad (6)$$

where

$\tilde{\Xi}_{11} = WA_{\lambda_i} + A_{\lambda_i}{}^T W - C^T \Gamma \Delta Q C + \varepsilon A_{\lambda_i}{}^T C^T J A_{\lambda_i}, \tilde{\Xi}_{12} = WB_{\lambda_i} + A_{\lambda_i}{}^T C^T P + \varepsilon A_{\lambda_i}{}^T C^T C B_{\lambda_i} - \frac{1}{2} J^T Q(\Gamma + \Delta$,

$\tilde{\Xi}_{22} = PCB_{\lambda_i} + B_{\lambda_i}{}^T C^T P - Q + \varepsilon B_{\lambda_i}{}^T C^T C B_{\lambda_i}$ then systems (3) is dichotomous for all f satisfying the sector condition.

Proof: By lemma 1, if system (3) has isolated equilibrium and there exist diagonal matrices P and Q with Q>0, a scalar $\varepsilon > 0$ and a symmetric matrix $W' = I_N \otimes W, Q' = I_N \otimes Q$ such that the following LMI is feasible

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ * & \Lambda_{22} \end{bmatrix} \le 0 \qquad (7)$$

where $\Lambda_{11} = W'(I_N \otimes A + L \otimes A_{12}) + (I_N \otimes A + L \otimes A_{12})^T W' - (I_N \otimes C^T \Gamma \Delta Q' C) + \varepsilon (I_N \otimes A + L \otimes A_{12})^T C^T C (I_N \otimes A + L \otimes A_{12})$

$$\Lambda_{12} = (I_N \otimes A + L \otimes A_{12})^T C^T P + \varepsilon (I_N \otimes A + L \otimes A_{12})^T C^T C (I_N \otimes B + L \otimes B_{12})$$
$$- \frac{1}{2} (I_N \otimes C)^T Q' (\Gamma + \Delta) + W'(I_N \otimes B + L \otimes B_{12})$$

$$\Lambda_{22} = PC(I_N \otimes B + L \otimes B_{12}) + (I_N \otimes B + L \otimes B_{12})^T C^T P - Q' + \varepsilon (I_N \otimes B + L \otimes B_{12})^T C^T C (I_N \otimes B + L \otimes B_{12})$$

Suppose U is an orthogonal matrix such that $U^T L U = \Lambda$, where $\Lambda = diag(\lambda_1, ..., \lambda_N)$, and λ_i are eigenvalues of L. Let $X = diag\{U \otimes I_n, U \otimes I_n\}$, and multiplying (7) on the left and on the right by X^T and X respectively. Than we can have (6), and this completes the proof.

In this section, network (3) with state feedback controller and uncertainties can be given as follows

$$\begin{aligned} \dot{x} &= (I_N \otimes (A + \Delta A) + L \otimes A_{12}) x + (I_N \otimes (B + \Delta B) + L \otimes B_{12}) u + B_1 u_k \\ z &= (I_N \otimes C) x \\ u &= -f(y) \end{aligned} \qquad (8)$$

where the uncertainties satisfy $\begin{bmatrix} \Delta A & \Delta B \end{bmatrix} = HF \begin{bmatrix} E_1 & E_2 \end{bmatrix}$, and $H \in R^{n \times i}, E_1 \in R^{j \times n}, E_2 \in R^{j \times m}$, $F^T F \le \lambda^2 I$ .The feedback controller chosen here is $u_{ki} = K x_i(t)$.

Theorem 2 Suppose that L is a matrix with $\lambda_i, i = 1,...N,$ are eigenvalues of L, different each other. If $A_{\lambda_i} = A + \lambda_i A_{12}$, $i = 1,...N$ have no eigenvalues on the imaginary axis, $(A_{\lambda_i}, B_{\lambda_i})$ is controllable, and (A_{λ_i}, C) (A_{λ_i}, C) is observable. And f satisfy the sector condition in (2). If system (8) has isolated equilibrium and there exist diagonal matrices P, Q>0, $\varepsilon > 0, \eta_1 > 0$, $\eta_2 > 0$,a invertible matrix X=X^T and matrix Y , such that the following LMI is feasible

$$\begin{bmatrix} M_{11} & M_{12} & M_{13} & H & \eta_1 X E_1^T & M_{16} & M_{17} \\ * & M_{22} & \varepsilon B^T C^T & PCH & \eta_1 E_2^T & 0 & 0 \\ * & * & -\varepsilon I & \varepsilon CH & 0 & 0 & 0 \\ * & * & * & -\eta_1 I & 0 & 0 & 0 \\ * & * & * & * & -\eta_1 I & 0 & 0 \\ * & * & * & * & * & -\eta_2 I & 0 \\ * & * & * & * & * & * & -\eta_2 I \end{bmatrix} \le 0 \tag{9}$$

where $M_{11} = A_{\lambda_i} X + X A_{\lambda_i}^T + B_1 Y + Y^T B_1^T, M_{12} = B_{\lambda_i} + X A_{\lambda_i}^T C^T P + Y^T B_1^T C^T P - \frac{1}{2} X C^T Q(\Gamma + \Delta)$,

$M_{13} = \varepsilon X A_{\lambda_i}^T C^T + \varepsilon Y^T B_1^T C^T$, $M_{16} = -\frac{1}{2} X C^T Q$,

$M_{17} = \eta_2 X C^T \Gamma \Delta, M_{22} = PCB_{\lambda_i} + B_{\lambda_i}^T C^T P - Q$. Then system (8) is dichotomy. And $K = YX^{-1}$ is the state feedback parameter matrix.

Proof: according to the theorem 1, system (8) is dichotomy, if the inequality (9) satisfy, then inequality (9) can be rewritten as

$$M + He\left\{ \begin{bmatrix} WH \\ PCH \\ \varepsilon CH \end{bmatrix} F \begin{bmatrix} E_1 & E_2 & 0 \end{bmatrix} \right\} \le 0 \tag{10}$$

where $M = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \varepsilon(A + \lambda_i A_{12})^T C^T \\ * & \Phi_{22} & \varepsilon B^T C^T \\ * & * & -\varepsilon I \end{bmatrix}$, $\Phi_{11} = W(A + \lambda_i A_{12}) + (A + \lambda_i A_{12})^T W - C^T \Gamma \Delta Q C$,

$\Phi_{12} = W(B + \lambda_i B_{12}) + (A + \lambda_i A_{12})^T C^T P - \frac{1}{2} C^T Q(\Gamma + \Delta)$, $\Phi_{22} = PC(B + \lambda_i B_{12}) + (B + \lambda_i B_{12})^T C^T P - Q$.

According to the lemma 2 and Schur complement lemma, the inequality (10) is satisfied, for $\eta_1 > 0$, $\eta_2 > 0$ if and only if the following inequality is satisfied

$$\begin{bmatrix} T_{11} & T_{12} & T_{13} & WH & \eta_1 E_1^T & T_{16} & T_{17} \\ * & T_{22} & \varepsilon B^T C^T & PCH & \eta_1 E_2^T & 0 & 0 \\ * & * & -\varepsilon I & \varepsilon CH & 0 & 0 & 0 \\ * & * & * & -\eta_1 I & 0 & 0 & 0 \\ * & * & * & * & -\eta_1 I & 0 & 0 \\ * & * & * & * & * & -\eta_2 I & 0 \\ * & * & * & * & * & * & -\eta_2 I \end{bmatrix} \le 0 \tag{11}$$

where $T_{11} = W(A + \lambda_i A_{12}) + (A + \lambda_i A_{12})^T W + W B_1 K + K^T B_1^T W, T_{12} = W(B + \lambda_i B_{12}) + (A + \lambda_i A_{12})^T C^T P$
$+ K^T B_1^T C^T P - \frac{1}{2} C^T Q(\Gamma + \Delta)$, $T_{13} = \varepsilon(A + \lambda_i A_{12})^T C^T + \varepsilon K^T B_1^T C^T$,

$T_{16} = -\frac{1}{2} C^T Q, T_{17} = \eta_2 C^T \Gamma \Delta$, $T_{22} = PC(B + \lambda_i B_{12}) + (B + \lambda_i B_{12})^T C^T P - Q$.

Let $Z = diag\{W^{-1}, I, \cdots, I\}$, and multiply inequality (11) on left and on right by Z. let $X = W^{-1}, Y = KW^{-1}$. Then the proof is completed.

Remark1: Inequality (9) is a nonlinear matrix inequality which involves $Y, P, X, Q, \varepsilon, \eta_1, \eta_2$. If P, Q, $\varepsilon, \eta_1, \eta_2$ are fixed, then inequality (9) is a linear matrix inequality. And if X, Y are fixed, inequality (9) is a linear matrix inequality which involves P, Q, $\varepsilon, \eta_1, \eta_2$. Then we use the alternating algorithm until the inequality feasible solution can be obtained.

Theorem 3 Suppose L is a matrix with $\lambda_i, i=1,\cdots,N$, which are eigenvalues of L, and are different from each other. Network (3) is absolutely stable for all f satisfying the sector condition in (2), if $A+\lambda_i A_{12}, i=1,\cdots,N$ are stable and there exist diagonal matrices P and Q with $Q\geq 0$, and symmetric matrices W, such that the following LMI is feasible

$$\begin{bmatrix} \tilde{\Lambda}_{11} & \tilde{\Lambda}_{12} \\ * & \tilde{\Lambda}_{22} \end{bmatrix} < 0 \tag{12}$$

where $\tilde{\Lambda}_{11} = W(A+\lambda_i A_{12}) + (A+\lambda_i A_{12})^T W - C^T \Gamma\Delta QC, \tilde{\Lambda}_{12} = W(B+\lambda_i B_{12}) + (A+\lambda_i A_{12})^T C^T P - \frac{1}{2} C^T Q(\Gamma+\Delta)$,

$\tilde{\Lambda}_{22} = PC(B+\lambda_i B_{12}) + (B+\lambda_i B_{12})^T C^T P - Q$.

Proof: By lemma 4, network (3) is absolutely stable, if $I_N \otimes A + L\otimes A_{12}$ is stable and there exist diagonal matrices P and $Q' = I_N \otimes Q$ with $Q\geq 0$, and symmetric matrices $W' = I_N \otimes W$ such that $\begin{bmatrix} \tilde{\Gamma} & \Psi \\ * & \Phi \end{bmatrix} < 0$, where

$\tilde{\Gamma} = W'(I_N \otimes A + L_1 \otimes A_{12}) + (I_N \otimes A + L\otimes A_{12})^T W' - (I_N \otimes C)^T \Gamma\Delta Q'(I_N \otimes C)$,

$\Phi = PC(I_N \otimes B + L_2 \otimes B_{12}) + (I_N \otimes B + L_2 \otimes B_{12})^T C^T P - Q'$,

$\Psi = W'(I_N \otimes B + L_2 \otimes B_{12}) + (I_N \otimes A + L\otimes A_{12})^T C^T P - \frac{1}{2}(I_N \otimes C)^T Q'(\Gamma+\Delta)$.

Let U be an orthogonal matrix such that $U^T LU = \Lambda$, where $\Lambda = diag(\lambda_1,\cdots,\lambda_N)$ and $\lambda_i, i=1,\cdots,N$ are eigenvalues of L. Take $X = diag(U\otimes I, U\otimes I)$. Multiplying on the left and the right of the above inequality by X^T and X, respectively, and this completes the proof.

Theorem 4 Suppose that L is a matrix, and $\lambda_i, i=1,\cdots,N$ are eigenvalues of L, different each other. Network (3) is absolutely stable for all f satisfying the sector condition in (2), if $A+\lambda_i A_{12}, i=1,\cdots,N$ are stable and there exist diagonal matrices P and Q with $Q\geq 0$, and matrice Y, $X = X^T$, such that the following inequality (13) satisfied.

Where $\Pi_{11} = (A+\lambda_i A_{12})X + X(A+\lambda_i A_{12})^T + Y^T B_1^T + B_1 Y, \Pi_{22} = PC(B+\lambda_i B_{12}) + (B+\lambda_i B_{12})^T C^T P - Q$.

Then system (3) is absolutely stable. $K = YX^{-1}$ is the controller matrix.

Proof: according to theorem 3, if the system (3) is absolute stable with controller $u_{ki} = Kx_i(t)$, then the inequality $\begin{pmatrix} \Lambda_{11} & \Lambda_{12} \\ * & \Lambda_{22} \end{pmatrix} < 0$ is satisfied, where $\Lambda_{11} = W(A+\lambda_i A_{12} + B_1 K) + (A+\lambda_i A_{12} + B_1 K)^T W - C^T \Gamma\Delta QC$,

$\Lambda_{22} = PC(B+\lambda_i B_{12}) + (B+\lambda_i B_{12})^T C^T P - Q$, and $\Lambda_{12} = W(B+\lambda_i B_{12}) + (A+\lambda_i A_{12} + B_1 K)^T C^T P - \frac{1}{2} C^T Q(\Gamma+\Delta)$.

Let $Z = diag\{W^{-1}, I,\cdots, I\}$. Multiply the inequality $\begin{pmatrix} \Lambda_{11} & \Lambda_{12} \\ * & \Lambda_{22} \end{pmatrix} < 0$ on the left and on the right by Z, let $X = W^{-1}, Y = KW^{-1}$, and using lemma 2, then we can get the inequality (13).

$$\begin{bmatrix}
\Pi_{11} & B+\lambda_i B_{12} & XC^T Q & \frac{1}{2}\eta_1 XC^T \Gamma\Delta & X(A+\lambda_i A_{12})^T & 0 & Y^T B_1^T & 0 & -0.5XC^T & 0 \\
* & \Pi_{22} & 0 & 0 & 0 & \eta_2 P^T C & 0 & \eta_3 P^T C & 0 & \eta_4 Q(\Gamma+\Delta) \\
* & * & -\eta_1 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -\eta_1 I & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -\eta_2 I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -\eta_2 I & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -\eta_3 I & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\eta_3 I & 0 & 0 \\
* & * & * & * & * & * & * & * & -\eta_4 I & 0 \\
* & * & * & * & * & * & * & * & * & -\eta_4 I
\end{bmatrix} < 0 \tag{13}$$

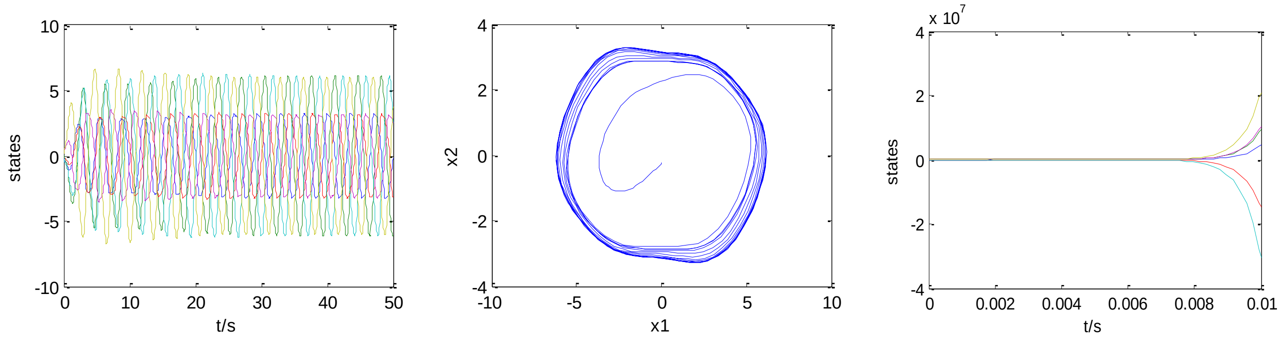
Remark 2: The dynamic consensus of system (3) is $\lim_{t\to\infty}\|x_i(t)-x_j(t)\|=0$, $\forall i, j=1,2...,N$, then theorem 4 gives the special results of dynamic consensus for network system (3) with $\lim_{t\to\infty}\|x_i(t)\|=\lim_{t\to\infty}\|x_j(t)\|=0$, $\forall i, j=1,2...,N$.

4 SIMULATION

In this section, the related examples are provided to validate the effectiveness of the results obtained above.

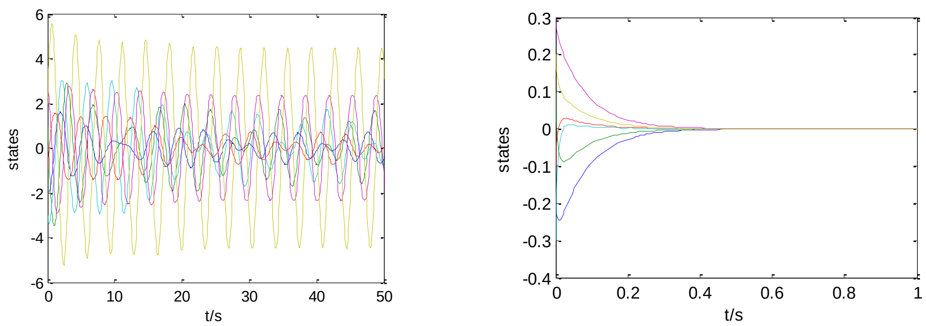
Example 1, for the network system (8) , which is composed by three subsystems, the related matrix of the network is given as $A=\begin{bmatrix}-0.4 & 3\\ -1 & -0.5\end{bmatrix}$, $B=\begin{bmatrix}1 & 0.5\\ 2.4 & 1\end{bmatrix}$, $C=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$, $B_1=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, $\Gamma=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$, $\Delta=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, $A_{12}=\begin{bmatrix}0.1 & 0.1\\ 0.1 & 0.1\end{bmatrix}$, $E_1=\begin{bmatrix}0.1 & 0.2\\ 0.2 & 0.1\end{bmatrix}$, $E_2=\begin{bmatrix}0.8 & 0\\ 0.2 & -0.5\end{bmatrix}$, $H=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix}$, $B_{12}=\begin{bmatrix}0.1 & 0.1\\ 0.1 & 0.1\end{bmatrix}$, $f_i(y_i)=\sin(y_i)$, $i=1,...,3$, $L=\begin{bmatrix}2 & -1 & -1\\ 0 & 1 & -1\\ -1 & -1 & 2\end{bmatrix}$. We can find that the eigenvalues of L are $0,\ 2,\ 3$.

When $x_0=(0.1,-0.2,-0.3,-0.1,0.2,0.3)$, the states diagram of uncertain network (8) without controller are shown in fig.1(a), and the phase diagram of $(x_1,\ x_2)$ of subsystem 1 is shown in fig.1(b). We can find that the network system (8) without controller contains limit circle. Using theorem 2, the controller K_1 =[1830.1 25.0;416.9 984.6] is obtained. The states diagram of network system (8) with the controller K_1 shown in fig.1(c), we can find that the system is dichotomous.



(a) (b) (c)

FIG.1:(A) STATE DIAGRAM OF NETWORK SYSTEM, (B) PHASE DIAGRAM OF SUBSYSTEM 1, (C) STATE DIAGRAM OF NETWORK SYSTEM (8) WITH CONTROLLER K_1



(a) (b)

FIG.2: (A) STATE DIAGRAM OF NETWORK SYSTEM WITH CONTROLLER K_1, (B) STATE DIAGRAM OF NETWORK SYSTEM (3) WITH CONTROLLER K_2

Example 2, for the network system (3), the nonlinear functions are chosen as $f_i(y_i)=0.5(1+\sin(y_i))$, $i=1,...,3$, then $\Gamma=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$, $\Delta=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ and the other parameter are chosen the same as example 1.

When $x_0 = (1, -2, -3, -1, 2, 3)$, the states diagram of network system (3) without controller are shown in fig.2(a), Using theorem 4, we have K_2=[-182.1389 68.7286; -75.1388 20.2610]. The states diagram of network system (3) with the controller K_2 shown in fig.2(b), we can find that the system is absolute stability.

5 CONCLUSIONS

In this paper, we consider the dichotomy and absolute stability of a class of nonlinear complex dynamical networks. The complex network system is composed of many subsystems by a certain interconnection relationship. The subsystems are a kind of nonlinear system, i.e., Lur'e systems. And the interconnection relationship of complex network systems referred in this paper not only concludes the interconnection of states, but also the interconnection of the sector nonlinear. Some necessary criterions are derived in this paper. Then, controller design methods of dichotomy and absolute stability of complex system are given and the related controller can be constructed via feasible solutions of a certain set of blinear matrix. At last, examples are given to illustrate the effectiveness of the proposed methods.

REFERENCES

[1] Wang X. F., Complex networks: Topology, dynamics, and synchronization. International Journal of Bifurcation and Chaos, 2002, vol.5, pp. 885-916

[2] MOYLAN JP and HILL J D, Stability criteria for large-scale systems, IEEE Transaction on Automatic Control, 1978, 23(2): 143-149

[3] WOLFE J D,CHICHKA D F, and SPEYER J L, Decentralized controller for unmanned aerial vehicle formation flight, USA: American Institute of Aeronautics and Astronautics, 1996

[4] I. Lestas, G. Vinnicombe, Scalable decentralized robust stability certificates for networks of interconnected heterogeneous dynamical systems,IEEE Trans, Automat. Control, 2006, vol. 51, pp. 1613-1635

[5] B.Bamieh, F.Paganini, M.Dahleh, Distributed control of spatially invariant systems, IEEE Trans. Automat. Control 2002, vol. 47, pp. 1091-1118

[6] Jinzhi Wang, Zhisheng Duan, Ying Yang, and Lin Huang, Analysis and control of Nonlinear systems with stationary sets: time-domain and frequency domain methods, Singapore: World Scientific Publisher, 2009

[7] Kai Wen, Zhiyong Geng, Stability of distributed heterogeneous systems with static nonlinear interconnections, System& Control , 2010, vol. 59, pp. 680- 686

[8] Liu X, Wang J. Z., Huang L., Global synchronization for a class of dynamical complex networks, Physica A, 2007, 386:543-556

[9] Zhisheng Duan, Jinzhi Wang. Stability analysis and decentralized control of a class of complex dynamical networks, Automatica, 2008, vol. 22, pp. 1028-1035

[10] Li Z. K., Duan Z. S., and G. R. Chen, Global consensus regions of multi-agent systems with nonlinear dynamics, Proceedings of WCICA, 2010, 580-585

[11] Li W., Chen Z., Liu Z., Leader-following formation control for second-order multiagent systems with time-varying delay and nonlinear dynamics, Nonlinear Dynamics, 2013:1-10

AUTHORS

[1]***Haijing Liu*** was born in Hebei, China, in 1988. She received the bachelor's degree in electronic information engineering from Hengshui University, Hengshui, China. She is a master in the college electronic information and control engineering in Beijing University of Technology. Her research interests include complex dynamical network systems, and robust control.

[2#]***Dun Ao*** was born in Inner Mongolia, China, in 1978. She received the Doctorate in general mechanics and mechanic foundation from Peking University, Beijing, China. She is the lecturer and master instructor in the college electronic information and control engineering in Beijing University of Technology. Her research interests include analysis and synthesis of complex systems, fuzzy control systems, and robust control. Email: aodun@bjut.edu.cn