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Fuzzy Reliability of Early Failure Mode in the Series System

Chenhui Liu, Jinhui Li, Yunfei Guo†

Mathematics Department, Yanbian University, Yanji, 133002, China

†Email: guoyunfei0413@sina.com

Abstract

In this paper, we analyze the fuzzy reliability of series system, focusing on the situation of the early failure mode. The early failure mode of fuzzy reliability mainly obeys the Weibull distribution. Based on this, we obtain the distribution function, density function, failure function, reliability function and the formula of mean time to failure. For the series system composed of multiple components, $C_1, C_2, \cdots, C_n$, especially when η=1 ,we also obtain the corresponding formulas. Finally, we cite an example to verify the formula and deepen the understanding.

Keywords: Series System; Early Failure Mode; Fuzzy Reliability; Weibull Distribution

1 INTRODUCTION

In 1965, based on the precise mathematical set theory, Zadeh gradually established operation, transformation rule and carried out the relevant theoretical research on the "fuzzy sets"[1]. In the domestic, Li Tingjie and Gao He conducted a preliminary study to fuzzy reliability and studied the relative concepts of fuzzy reliability[2].In 1994, Zhang Shuhua and Yu Hongbo applied the principle and method of fuzzy mathematics to set up a series of formula to calculate the fuzzy reliability of series system, and computed an early failure type of the model[3]. This paper analyses the distribution of the early failure mode in the series system. And the corresponding formulas are also provided. Zhang Shuhua and Yu Hongbo's paper analyzed a special case of early failure type namely η=1, $m = 1/2$ .In this paper, we generalize the type namely $m < 1$.

2 PRE- KNOWLEDGE

Series system is a system that only can be effective when all the components are working. Graphics are as follows.

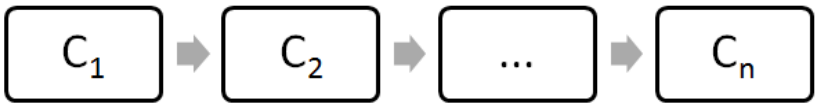


FIG. 1 THE SERIES SYSTEM

Definition 2.1[4] Li Tingjie and Gao He defined the following symbols:

C : System works normally

C_j :The j_{th} part works normally

$\underset{\sim}{A}_i$: Fuzzy state set

They also obtained the following formulas:

$$C = \bigcap_{j=1}^{n} C_j \tag{2.1}$$

If the failure of the parts is independent of each other, then

$$\begin{cases} \text{The system of common failure rate} & P(\mathrm{C}) = \prod_{j=1}^{n} P(\mathrm{C}_j) \\ \text{The system of fuzzy failure rate} & P(\mathrm{C}\Delta\tilde{A}_i) = \mathrm{P}(\tilde{A}_i \mid C)\bullet\mathrm{P}(\mathrm{C}) \\ \text{The part of fuzzy failure rate} & P(\mathrm{C}_j\,\Delta\tilde{A}_i) = \mathrm{P}(\tilde{A}_i \mid C_j)\bullet\mathrm{P}(\mathrm{C}_j) \end{cases} \tag{2.2}$$

Definition 2.2[4] According to the general reliability theory and literature:

$$\begin{array}{l} P(\mathrm{C}) = R_s, P(\mathrm{C}_j) = \mathrm{R}_j \\ \mathrm{P}(\mathrm{C}\Delta\tilde{A}_i) = \tilde{R}_s, P(\mathrm{C}_j\,\Delta\tilde{A}_i) = \tilde{R}_j \\ \mathrm{P}(\mathrm{A}_i \mid \mathrm{C}) = \mu_{\tilde{A}_i}(\mathrm{R}_s), \mathrm{P}(\tilde{A}_i \mid C_j) = \mu_{\tilde{A}_i}(\mathrm{R}_j) \end{array} \tag{2.3}$$

Lemma 2.1[5] The failure distribution of series system obeys Weibull distribution $Wei(m,\eta)$. Weibull distribution's two parameters are .Among them, η is referred to the characteristic of life(the distribution of the 0.632 digits), m is referred to the shape parameter and $\eta > 0, m > 0$. $Wei(m,\eta)$'s information is the following.

Distribution function:

$$F(t) = 1 - \exp\left\{-(\frac{t}{\eta})^m\right\} \qquad t > 0 \tag{2.4}$$

Density function:

$$\mathrm{f(t)} = [F(\mathrm{t}]' = \frac{m}{\eta}(\frac{t}{\eta})^{m-1} \cdot \exp\left\{-(\frac{t}{\eta})^m\right\} \qquad t > 0 \tag{2.5}$$

Failure function:

$$\lambda(\mathrm{t}) = \frac{f(\mathrm{t})}{1 - F(\mathrm{t})} = \frac{m}{\eta}(\frac{t}{\eta})^{m-1} \qquad t > 0 \tag{2.6}$$

Lemma 2.2 According to 2.4,2.5,2.6 above, we can obtain that

The reliability function :

$$R(\mathrm{t}) = 1 - F(\mathrm{T}) = \exp\left\{-(\frac{t}{\eta})^m\right\} \qquad \mathrm{t} > 0 \tag{2.7}$$

The mean time to failure :

$$MTTF = \frac{1}{\lambda(\mathrm{t})} = \frac{1}{\frac{m}{\eta}(\frac{t}{\eta})^{m-1}} = \frac{\eta}{m(\frac{t}{\eta})^{m-1}} \qquad t > 0 \tag{2.8}$$

3 FUZZY RELIABILITY ANALYSIS IN EARLY FAILURE MODE

According to the Lemma 2.1, when $m < 1$, it is early failure mode.

For a series system composed of n parts, $C_1, C_2, \bullet\bullet\bullet, C_n$, the analysis of fuzzy reliability is the following.

Assume that the failure of each part is independent of each other, and it obeys $Wei(m,\eta)$, and also let $\eta = 1$, then

Theorem 3.1 The fuzzy reliability function of system:

$$\tilde{R}_s(\mathrm{t}) = \mu_{\tilde{A}_i}(\mathrm{R}_s)\bullet\exp[-\sum_{j=1}^{n} \mathrm{t}^{m_j}] \qquad t > 0 \tag{3.1}$$

Proof. First we should infer the common reliability function of system:

$$\begin{aligned} \mathrm{R}_s(\mathrm{t}) &= \prod_{j=1}^{n} R_j(\mathrm{t}) \\ &= \exp(-\mathrm{t}^{m_1}) \times \exp(-\mathrm{t}^{m_2}) \times \cdots \times \exp(-\mathrm{t}^{m_n}) \\ &= \exp[-(\mathrm{t}^{m_1} + \mathrm{t}^{m_2} + \cdots \mathrm{t}^{m_n})] \\ &= \exp[-\sum_{j=1}^{n} \mathrm{t}^{m_j}] \qquad t > 0 \end{aligned} \tag{3.2}$$

Then plug this formula into the following formula, we obtain

$$\underset{\sim}{\mathrm{R}}_s(\mathrm{t}) = \mu_{\underset{\sim}{A}_i}(\mathrm{R}_s) \bullet \mathrm{R}_s(\mathrm{t}) = \mu_{\underset{\sim}{A}_i}(\mathrm{R}_s) \bullet \exp[-\sum_{j=1}^{n} \mathrm{t}^{m_j}] \qquad t > 0 \tag{3.3}$$

Collary 3.1 If $m_j = m,\ j = 1, 2, \cdots, n$, then

$$\underset{\sim}{\mathrm{R}}_s(\mathrm{t}) = \mu_{\underset{\sim}{A}_i}(\mathrm{R}_s) \bullet [\exp(-\mathrm{t}^{m})]^{n} \qquad t > 0 \tag{3.4}$$

Theorem 3.2 The fuzzy failure function of system:

$$\underset{\sim}{\lambda}_s(\mathrm{t}) == \sum_{j=1}^{n} m_j(\mathrm{t})^{m_j - 1} - \frac{d\,\mu_{\underset{\sim}{A}_i}(R_s)}{\mu_{\underset{\sim}{A}_i}(R_s)\,dt} \qquad t > 0 \tag{3.5}$$

Proof. First we should infer the common failure function of system:

$$\begin{aligned} \lambda_s(\mathrm{t}) &= \lambda_1(\mathrm{t}) + \lambda_2(\mathrm{t}) + \cdots + \lambda_n(\mathrm{t}) \\ &= \sum_{j=1}^{n} \lambda_j(\mathrm{t}) = \sum_{j=1}^{n} m_j(\mathrm{t})^{m_j - 1} \qquad t > 0 \end{aligned} \tag{3.6}$$

Then plug this formula into the following formula, we obtain

$$\begin{aligned} \underset{\sim}{\lambda}_s(\mathrm{t}) &= -\frac{d\underset{\sim}{R}_s}{\underset{\sim}{R}_s dt} = -\frac{d\left[\mu_{\underset{\sim}{A}_i}(R_s) \bullet \mathrm{R}_s\right]}{\mathrm{R}_s \bullet \mu_{\underset{\sim}{A}_i}(R_s)\,dt} = \frac{-\mathrm{R}_s\, d\,\mu_{\underset{\sim}{A}_i}(R_s) - \mu_{\underset{\sim}{A}_i}(R_s)\, d\,\mathrm{R}_s}{\mu_{\underset{\sim}{A}_i}(R_s) \bullet \mathrm{R}_s\, dt} \\ &= \frac{-dR_s}{R_s dt} - \frac{d\,\mu_{\underset{\sim}{A}_i}(R_s)}{\mu_{\underset{\sim}{A}_i}(R_s)\,dt} = \lambda_s - \frac{d\,\mu_{\underset{\sim}{A}_i}(R_s)}{\mu_{\underset{\sim}{A}_i}(R_s)\,dt} \\ &= \sum_{j=1}^{n} m_j(\mathrm{t})^{m_j - 1} - \frac{d\,\mu_{\underset{\sim}{A}_i}(R_s)}{\mu_{\underset{\sim}{A}_i}(R_s)\,dt} \qquad t > 0 \end{aligned} \tag{3.7}$$

Collary 3.2 If $m_j = m,\ j = 1, 2, \cdots, n$, then

$$\underset{\sim}{\lambda}_s(\mathrm{t}) = n \times m(\mathrm{t})^{m-1} - \frac{d\,\mu_{\underset{\sim}{A}_i}(R_s)}{\mu_{\underset{\sim}{A}_i}(R_s)\,dt} \qquad t > 0 \tag{3.8}$$

Theorem 3.3 The fuzzy system:

$$\underset{\sim}{MTTF}_s = \frac{1}{\sum_{j=1}^{n} m_j(\mathrm{t})^{m_j - 1} - \frac{d\,\mu_{\underset{\sim}{A}_i}(R_s)}{\mu_{\underset{\sim}{A}_i}(R_s)\,dt}} \qquad t > 0 \tag{3.9}$$

Proof. First we should infer the mean time to failure of system:

$$MTTF_s = \frac{1}{\lambda_s(\mathrm{t})} = \frac{1}{\sum_{j=1}^{n} m_j(\mathrm{t})^{m_j - 1}} \qquad t > 0 \tag{3.10}$$

Then plug this formula into the following formula, we obtain

$$\underset{\sim}{MTTF}_s = \frac{1}{\underset{\sim}{\lambda}_s(t)} = \frac{1}{\sum_{j=1}^{n} m_j (t)^{m_j - 1} - \frac{d\underset{\sim}{\mu}_{A_1}(R_s)}{\underset{\sim}{\mu}_{A_1}(R_s)dt}} \qquad t > 0 \tag{3.11}$$

Collary 3.3 If $m_j = m,\ j = 1, 2, \cdots, n$,then

$$\underset{\sim}{MTTF}_s = \frac{1}{\underset{\sim}{\lambda}_s(t)} = \frac{1}{n \times m(t)^{m-1} - \frac{d\underset{\sim}{\mu}_{A_1}(R_s)}{\underset{\sim}{\mu}_{A_1}(R_s)dt}} \qquad t > 0 \tag{3.12}$$

4 EXAMPLE

Put electrical appliances into series. Assume that the system is at the early stage of use to solve the very reliable fuzzy reliability and fuzzy failure rate of the series system in 6 minutes. Let $\underset{\sim}{A}_1$: be in highly reliable state. Given that:

$$\mu_{\underset{\sim}{A}_1}(0.378) = 0.7 \tag{4.1}$$

see graphic below[3]:

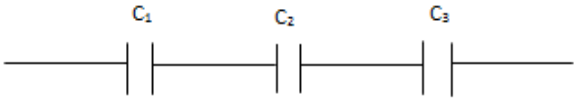


FIG. 2 THE SERIES SYSTEM

Because the elements belong to the early using, the failure rate function is the early failure type.

Among them $m = \frac{1}{2}$, in Zhang Shuhua's paper, she has already done it, but she has made a mistake, she did it wrong. According to (2.5), let η=1,$m = \frac{1}{3}$, then we can obtain that

$$f(t) = \frac{1}{2}(t)^{-\frac{1}{3}} \cdot \exp\left\{-(t)^{\frac{1}{3}}\right\} \qquad t > 0 \tag{4.2}$$

Because t=6min=0.1h, $\underset{\sim}{A}_1$: highly reliable state, R_s =0.378 , $\mu_{\underset{\sim}{A}_1}(0.378) = 0.7$, according to (3.1.2), then

$$\begin{aligned} \underset{\sim}{R}_s(t) &= \mu_{\underset{\sim}{A}_1}(0.378) \cdot \exp[-3t^{\frac{1}{3}}] \\ &= 0.7 \cdot \exp[-3t^{\frac{1}{3}}] \\ &= 0.01618 \qquad t > 0 \end{aligned} \tag{4.3}$$

Because $\mu_{\underset{\sim}{A}_1}(R_s)$ has been constant, according to (3.1.4), then

$$\begin{aligned} \underset{\sim}{\lambda}_s(t) &= n \times m(t)^{m-1} - \frac{d\underset{\sim}{\mu}_{A_1}(R_s)}{\underset{\sim}{\mu}_{A_1}(R_s)dt} \\ &= 3 \times \frac{1}{3} \times 0.1^{-\frac{1}{3}} - 0 \\ &= 2.1544 \qquad t > 0 \end{aligned} \tag{4.4}$$

$$\underset{\sim}{MTTF}_s = \frac{1}{\underset{\sim}{\lambda}_s(t)} = \frac{1}{2.1544} = 0.4642 \qquad t > 0 \tag{4.5}$$

So the highly reliable fuzzy reliability is 0.01618,the fuzzy of mean to failure is 0.4642h.

5 CONCLUSIONS

In this paper, we extend Zhang Shuhua's paper, having concluded a series of formulas about fuzzy reliability of early

failure mode in the series system. We know that early failure mode obeys Weibull distribution.

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AUTHORS



[1]**Chenhui Liu** was was born on July 11th, 1995 in HeNan province. She is studying in Yanbian University.



[2]**Jinhui Li** was was born on March 20th, 1993 in JiangXi province. She is studying in Yanbian University.



[3]**Yunfei Guo** was born on April 13th, 1983 in Jilin province, and received his M.S. degrees in Yanbian University, China in 2010. He is a Lecture of Yanbian University. His research interests are reliability and statistical analysis.