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Stability Analysis of a Kind of Repairable Control System with Partial State

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Abstract

A kind of repairable control system with partial state is investigated by functional analysis method and linear semigroup theory in this paper. We prove the existence of the eigenvalue 0. Moreover, we analyze the resolvent of the system operator and obtain the distribution of spectrum points of the system operator. Then the stability of the system is obtained.

Keywords: Spectrum; Repairable System; Resolvent; Stability

1 INTRODUCTION

Nowadays, reliability theory is more and more important in many industries. And repairable system has been studied by many researchers through various models. This system model is one of them. According to [1], there are four states that the system can be in.

(I) Normal State- This state is defined as the functioning of the system at its full capacity.

(II) Partial State- This state is defined as the functioning of the system, say tolerable, counting 70% of its full capacity (due to some component failure).

(III) Complete Failure- This state is defined as the total failure where a system does not operate at all or operates between 0 and 70% of its full capacity.

(IV) Catastrophic Failure- This state is defined as failure where a system or equipment carries out some unacceptable operations.

In [1], Ballbirs Dhillon developed a mathematical model with the repair from its complete failure and catastrophic failure modes. Also he proved the existence of the solution of the system by Laplace transform; In [2], Baolin Feng proved the existence of a unique non-negative solution of the system; In [3], a simplified model of this system is studied. The distribution of the spectrum of the simplified model is analysed and the stability of the system is obtained. Here we will analyse the distribution of spectrum points of the system operator and then we obtain the stability of the system.

According to [1], the state diagram of the system is shown in FIG.1:

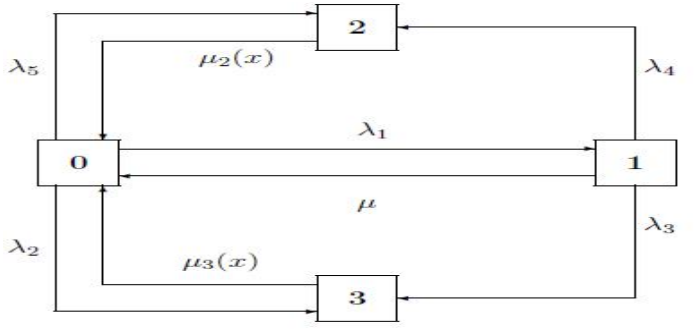


FIG. 1 STATE TRANSITION DIAGRAM

This system model can be expressed by a group of equations:

$$\begin{cases} \dfrac{dp_0(t)}{dt} = -(\lambda_1+\lambda_2+\lambda_5)p_0(t) + \mu p_1(t) + \sum\limits_{j=2}^{3}\int_0^{\infty} p_j(x,t)\mu_j(x)dx \\ \dfrac{dp_1(t)}{dt} = -(\lambda_3+\lambda_4+\mu)p_1(t) + \lambda_1 p_0(t) \\ \dfrac{\partial p_j(x,t)}{\partial t} + \dfrac{\partial p_j(x,t)}{\partial x} = -\mu_j(x)p_j(x,t), \quad (j=2,3) \\ p_2(0,t) = \lambda_4 p_1(t) + \lambda_5 p_0(t) \\ p_3(0,t) = \lambda_3 p_1(t) + \lambda_2 p_0(t) \\ p_0(0) = 1, p_i(0) = p_2(x,0) = p_3(x,0) = 0, \quad (i=0,1) \end{cases} \tag{1.1}$$

For brevity, let $a_0 = \lambda_1+\lambda_2+\lambda_5$; $a_1 = \lambda_3+\lambda_4+\mu$; Here i represents the i -th state of the system; $i=0$ represents the system is in its normal mode state; $i=1$ represents the system is in its partial failure mode state; $i=2$ represents the system is in its complete failure mode state; $i=3$ represents the system is in its catastrophic failure mode state; t represents the time; λ_s represents the constant failure rate of the system, for $s=1,2,3,4,5$; $p_i(t)$ represents the probability that the system is in state i at time t for $i=0,1$; $p_i(x,t)$ represents the probability density (with respect to repair time) that the failed system is in state j and has an elapsed repair time of x for $j=2,3$; $\mu_i(x)$ represents the repair rate when the failed system is in state j and has an elapsed repair time of x for $j=2,3$.

Considering the actual background, we suppose that:

$$0 \le \mu(x) < +\infty, \quad 0 \le x < +\infty, \quad \int_0^{\infty}\mu_j(\xi)d\xi = \infty, \quad \lim_{x\to\infty}\frac{1}{x}\int_0^{x}\mu_j(\xi)d\xi < +\infty.$$

We choose the state space as

$$X = C^2 \times L^1[0,\infty) \times L^1[0,\infty) \times L^1[0,\infty),$$

For $y = (y_0, y_1, y_2(x), y_3(x)) \in X$, We define its norm as follow：

$$\|y\| = |y_0| + |y_1| + \|y_2\|_{L^1} + \|y_3\|_{L^1}.$$

It is obvious that $(X, \|\cdot\|)$ is a Banach space.

We define operator A , B and their domains:

$$A = diag\left(-a_0, -a_1, -\frac{d}{dx} - \mu_2(x), -\frac{d}{dx} - \mu_3(x)\right),$$

$$D(A) = \left\{ \vec{P} \in X \,\middle|\, \frac{dp_j(x)}{dx} \in L^1(R^+), j=2,3, p_2(0) = \lambda_4 p_1(t) + \lambda_5 p_0(t), p_3(0) = \lambda_3 p_1(t) + \lambda_2 p_0(t) \right\}.$$

$$B = \begin{pmatrix} 0 & \mu & \int_0^{\infty} \bullet \mu_2(x)dx & \int_0^{\infty} \bullet \mu_3(x)dx \\ \lambda_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

and $D(B) = X$.

Then the system (1.1) can be expressed as an Abstract Cauchy Problem in Banach space X :

$$\begin{cases} \dfrac{d\vec{P}(t)}{dt} = (A+B)\vec{P}(t), t \ge 0 \\ \vec{P}(0) = (1,0,0,0)^T \end{cases} \tag{1.2}$$

2 SPECTRUM DISTRIBUTION AND STABILITY ANALYSIS OF THE SYSTEM

2.1 Theorem

0 is a simple eigenvalue of the operator of the system..

Proof:

We consider $(\gamma I - A - B)P = 0$ as the following equations: equation:

$$\begin{cases} (\gamma + a_0)p_0 - \mu_1 p_1 - \sum_{j=2}^{3}\int_0^{\infty} p_j(x)\mu_j(x)dx = 0; \\ -\lambda_1 p_0 + (\gamma + a_1)p_1 = 0; \\ \dfrac{dp_j(x)}{dx} + (\gamma + \mu_j(x))p_j(x) = 0, j = 2,3; \\ p_2(0)=\lambda_4 p_1+\lambda_5 p_0 \\ p_3(0)=\lambda_3 p_1+\lambda_2 p_0 \end{cases} \tag{2.1}$$

Solving (2.1), then we can get that:

$$p_j(x) = p_j(0)e^{-\int_0^x(\gamma+\mu_j(\xi))d\xi}, j = 2,3.$$

Combining it into the first equation of (2.1), we can obtain that:

$$(\gamma + a_0)p_0 - \mu_1 p_1 - \sum_{j=2}^{3} p_j(0)\int_0^{\infty}\mu_j(x)e^{-\int_0^x(\gamma+\mu_j(\xi))d\xi}dx = 0.$$

Then we have：

$$\begin{cases} (\gamma + a_0)p_0 - \mu_1 p_1 - \sum_{j=2}^{3} p_j(0)\mu_{j,\gamma} = 0 \\ \lambda_1 p_0 - (\gamma + a_1)p_1 = 0 \\ \lambda_4 p_1 + \lambda_5 p_0 - p_2(0) = 0 \\ \lambda_3 p_1 + \lambda_2 p_0 - p_3(0) = 0 \end{cases} \tag{2.2}$$

Where, $\mu_{j,\gamma} = \int_0^{\infty}\mu_j(x)e^{-\int_0(\gamma+\mu_j(\xi))d\xi}dx, j = 2,3$

The system determinant can be written as $D(\gamma)$， then

$$D(\gamma) = \begin{vmatrix} \gamma + a_0 & -\mu & -\mu_{2,\gamma} & -\mu_{3,\gamma} \\ \lambda_1 & -(\gamma + a_1) & 0 & 0 \\ \lambda_5 & \lambda_4 & -1 & 0 \\ \lambda_2 & \lambda_3 & 0 & -1 \end{vmatrix}$$

Then it can be easily found that if $\gamma \in C$ is an eigenvalue of the operator $A + B$, we have $D(\gamma) = 0$. Contrary, if there is $\gamma \in C$ such that $D(\gamma) = 0$, then the equations have non-zero solution $(p_0, p_1, p_2(x), p_3(x))$. So,

$$\left(p_0, p_1, p_2(0)e^{-\int_0^x(\gamma+\mu_2(\xi))d\xi}, p_3(0)e^{-\int_0^x(\gamma+\mu_3(\xi))d\xi}\right) \in D(A + B)$$

is a solution of (2.1).

In particular, when $\gamma = 0$,

$$\mu_{j,\gamma} = \int_0^{\infty}\mu_j(x)e^{-\int_0^x(\gamma+\mu_j(\xi))d\xi}dx = 1, j = 2,3.$$

So, $D(\gamma)=0$. From that we can know $\gamma=0$ is an eigenvalue of operator $A+B$ and the eigenvector corresponding to eigenvalue 0 is $\vec{P}=(p_0,p_1,p_2(x),p_3(x))$. Here

$$\begin{cases} p_1=\dfrac{\lambda_1}{a_1}p_0 \\ p_2(x)=\left(\dfrac{\lambda_4\lambda_1}{a_1}+\lambda_5\right)p_0e^{-\int_0^x\mu_3(\xi)d\xi} \\ p_3(x)=\left(\dfrac{\lambda_3\lambda_1}{a_1}+\lambda_2\right)p_0e^{-\int_0^x\mu_3(\xi)d\xi} \end{cases}$$

Let $\vec{Q}=(1,1,1,1)$, we have $<\vec{P},\vec{Q}>=p_0+p_1+\int_0^\infty p_2(x)dx+\int_0^\infty p_3(x)dx>0$. For any $\vec{P}\in D(A+B)$, we have $<(A+B)\vec{P},\vec{Q}>=0$. That is $(A+B)^*\vec{Q}=0$. So 0 is a simple eigenvalue of operator $A+B$

2.2 Theorem

Let $\rho(A+B)$ denote the resolvent set of $A+B$, then $\left\{\gamma\in C\middle|\operatorname{Re}\gamma>0 \ or\ \gamma=ia,a\in R,a\neq0\right\}\subset\rho(A+B)$

Proof:

For $\operatorname{Re}\gamma>0 \ or\ \gamma=ia,a\in R,a\neq0$ and $\forall\vec{Y}=\left(y_0,y_1,y_2(x),y_3(x)\right)$, we consider the equation $\left[\gamma I-(A+B)\right]\vec{P}=\vec{Y}$.

$$\begin{cases} (\gamma+a_0)p_0-\mu_1p_1-\sum_{j=2}^{3}\int_0^\infty p_j(x)\mu_j(x)dx=y_0 \\ -\lambda_1p_0+(\gamma+a_1)p_1=y_1 \\ \dfrac{dp_j(x)}{dx}+(\gamma+\mu_j(x))p_j(x)=y_j(x),j=2,3. \\ p_2(0)=\lambda_4p_1+\lambda_5p_0 \\ p_3(0)=\lambda_3p_1+\lambda_2p_0 \end{cases} \quad (2.3)$$

Solving (2.3), we have

$$p_j(x)=p_j(0)e^{-\int_0^x[\gamma+\mu_i(\xi)]d\xi}+\int_0^x e^{-\int_\tau^x[\gamma+\mu_i(\xi)]}y_j(\tau)d\tau \quad (2.4)$$

By the lemma 2 and lemma 3 in [4], the following inequalities hold:

$$\int_0^\infty\left|e^{-\int_0^x[\gamma+\mu_j(\xi)]d\xi}\right|dx\le\int_0^\infty\left|e^{-\int_0^x\mu_j(\xi)]d\xi}\right|dx\le N_1$$

$$\int_0^\infty\left|\int_0^x e^{-\int_\tau^x[\gamma+\mu_j(\xi)]}y_j(\tau)d\tau\right|dx\le\int_0^\infty dx\int_0^\infty\left|e^{-\int_\tau^x[\gamma+\mu_j(\xi)]}\right|\left|y_j(\tau)\right|d\tau\le\int_0^\infty\left|y_j(\tau)\right|dx\int_\tau^\infty e^{-\int_\tau^x[\mu_j(\xi)]d\xi}dx\le\left\|y_j\right\|_{L^1[0,\infty)}N_2$$

Combining (2.4) into (2.3), we have

$$\begin{cases} (\gamma+a_0)p_0-(\lambda_5b_2+\lambda_2b_3)p_0-(\mu+\lambda_4b_2+\lambda_3b_3)p_1=G_2+G_3+y_0 \\ -\lambda_1p_0+(\gamma+a_1)p_1=y_1 \end{cases} \quad (2.5)$$

Where

$$b_j=\int_0^\infty\mu_j(x)e^{-\int_0^x[\gamma+\mu_j(\xi)]d\xi}dx,\quad G_j=\int_0^\infty\mu_j(x)\int_0^x\left|e^{-\int_\tau^x[\gamma+\mu_j(\xi)]}\right|y_j(\tau)d\tau dx,\quad j=2,3$$

So the coefficient matrix of (2.5) is $D=\begin{pmatrix}\gamma+a_0-\lambda_5b_2-\lambda_2b_3 & -\mu-\lambda_4b_2-\lambda_3b_3\\ -\lambda_1 & \gamma+a_1\end{pmatrix}$

When $\mathrm{Re}\gamma > 0$ *or* $\gamma = ia, a \in R, a \neq 0$, it is obviously that $|b_i| \leq 1$. From the method of [5], the coefficient matrix is strictly diagonally dominant and $\det(D) \neq 0$ .So the equation (2.5) has the unique solution. It implies that $[\gamma I - (A+B)]$ is a surjective. As $[\gamma I - (A+B)]$ is closed and $D(A+B)$ is dense in X , according to inverse operator theorem, we have $[\gamma I - (A+B)]^{-1}$ is a bounded linear operator. So $\gamma \in \rho(A+B)$ for $\mathrm{Re}\gamma > 0$ *or* $\gamma = ia, a \in R, a \neq 0$. It implies that the spectrum points of the system operator except 0 are on the left semi-plane. So the system is asymptotic stable.

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